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The Intensity Distribution in the Continuous Spectrum and the Intensities of the Hydrogen Lines in γ Cassiopeiæ. By H. H. Plaskett. (Plates 15, 16.)

(*Communicated by the Director, Dominion Astrophysical Observatory, Victoria, B.C.*)

Though much is now known of the origin and conditions of production of the emission and absorption lines in stellar spectra, comparatively little work has been done on the energy distribution in the continuous background. The reason for this is not far to seek. For to the inherent difficulties of the astronomical problem —the feebleness of the source and the variability of the atmospheric transmission—have been added the limitations of the methods hitherto available for the determination of spectral intensities. A method, however, has recently been devised by T. R. Merton and J. W. Nicholson which, by reason of its accuracy and simplicity, seems particularly suited to a study of the intensity distribution in stellar spectra.

This paper contains an account of a preliminary application of this method to the star γ Cassiopeiæ. This star is of photographic magnitude 2·01, type B 0 *p*, and shows bright hydrogen lines. A considerable amount of work has been done on its spectrum, notably by Lockyer, Sidgreaves, and R. H. Curtiss. A brief description is given of the method, as the original papers are not readily available to some, and an account of the necessary modifications to adopt the method to stellar spectroscopy. There follow the results showing the relative intensities on an absolute scale of the spectrum between 0·42μ and 0·51μ, and finally a comparison of the intensity decrement in the Balmer series as displayed in the star and in the laboratory.

1. *The Method.*

The photographic plate, as the photometric receiver in the determination of spectral intensities, possesses great advantages

over the eye, or such electrical arrangements as the bolometer, the thermo-couple, the photo-electric cell, etc. For it permits the registration of the intensities in the whole visible spectrum at the same time, and is not limited to bright sources of light. Its use as a photometric receiver, however, is open to two serious objections. In the first place the density of the image on the photographic plate is not related in any simple way to either the intensity of the source or the duration of the exposure. Two sources, then, can be regarded of the same intensity only when they produce, after equal exposures, images of the same density on adjacent portions of the same photographic plate.* In the second place the photographic plate is not equally sensitive to light of all colours; nor is its colour curve constant, but varies in an irregular way with the make of the plate, and even with different batches of plates by the same maker. It will be noted that the method to be described, while it possesses the obvious advantages of the photographic plate, is at the same time so arranged as to overcome its inherent defects.

The essential features in Merton and Nicholson's method † are the use of a neutral tint wedge, the calibration of the photographic plate by the spectrum of the positive crater of the carbon arc, and the elimination of personal errors in measurement by the enlargement of the plate through a "ruled" process screen. They have applied this method to the determination of the line intensities in the spectra of hydrogen and helium, and also to a determination of the intensity distribution in individual lines.

In front of the slit of the spectroscope is placed a wedge of neutral tint glass, to which is cemented a piece of clear glass so as to make a parallel plate. The light from the source, uniformly distributed along the length of the slit, is reduced in intensity by an amount depending upon the thickness of the wedge which it traverses. Thus upon the photographic plate are imprinted the usual lines, which, being images of the slit, will be attenuated towards the thick end of the wedge, and will therefore be of different heights depending upon their original intensities. On account of the difficulty of determining the exact point at which a line ends, a negative enlargement of the plate is made through a "ruled" process screen. On the enlargement each line is now made up of dots, and it is a comparatively easy process to pick out the last dot which is affected, and hence to measure the height of the line.

According to a well-known law of absorption, light of intensity I_0 and wave-length λ falling upon an absorbing material of thickness y is transmitted with intensity I where

$$I = I_0 10^{-\rho_\lambda y},$$

* J. Hartmann, *Ap. J.*, **10**, 321.

† *Roy. Soc. Phil. Trans.*, **217**, 237; **216**, 459.

ρ_λ being the coefficient of extinction of the material which varies with λ. In fig. 1, if I_1 is the light transmitted by the thin edge of the wedge, I_2 the light transmitted by the thick end, then

$$I_1 = I_0 10^{-\rho_\lambda \delta} \quad \text{and} \quad I_2 = I_0 10^{-\rho_\lambda (l \tan \alpha + \delta)},$$

where δ is the thickness of the thin edge of the wedge, l is the length of the wedge, and α its angle. The ratio of transmission thick : thin is given by

$$I_2/I_1 = 10^{-\rho_\lambda l \tan \alpha} \quad \text{and} \quad \rho_\lambda l \tan \alpha = -\log 10(I_2/I_1) = D_\lambda.$$

D_λ by analogy to the definition of density, is termed the density of the wedge. ρ_λ and D_λ are wedge constants, which depend upon the nature of the neutral tint glass and the dimensions of the wedge. Their values are readily determined (see sec. 2).

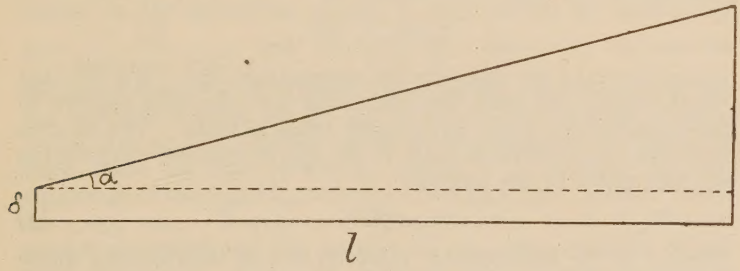

Fig. 1.

If radiation of wave-length λ and intensity I_λ falls on the wedge, a certain thickness y will reduce it to an intensity I_c. I_c is the intensity which is just photographically visible, or, in terms of the process screen enlargement, the last dot that can be seen.

$$I_c = I_\lambda 10^{-\rho_\lambda y} \quad \text{and} \quad y = l_\lambda \tan \alpha + \delta \quad . \quad . \quad (1)$$

where l_λ is the distance from the thin end of the wedge corresponding to the thickness y. After enlargement l_λ becomes h_λ where $l_\lambda/h_\lambda = l/H$, H being the length of the wedge after enlargement. Hence from equation (1)

$$I_\lambda/I_c = 10^{\rho_\lambda (l_\lambda \tan \alpha + \delta)} \quad \therefore \quad \log I_\lambda/I_c = \rho_\lambda (l_\lambda \tan \alpha + \delta)$$

$$= \rho_\lambda l \tan \alpha \cdot \frac{h_\lambda}{H} + \rho_\lambda \delta = D_\lambda h_\lambda / H + \rho_\lambda \delta.$$

Therefore the intensity of the line is given by

$$I_\lambda/I_c = \log_{10}{}^{-1}(D_\lambda h_\lambda/H + \rho_\lambda \delta) \quad . \quad . \quad . \quad (2)$$

where the notation $\log_{10}{}^{-1}$ signifies an anti-logarithm. Since I_c is an unknown function of λ (on account of the colour curve of

the plate) and the exposure time, equation (2) will give only the photographic intensities of the lines in a spectrum relative to some line arbitrarily chosen as unity.

In order to determine the intensities as given by equation (2) on an absolute scale, an exposure is made, on at least one of each batch of plates, of the spectrum given by the positive crater of the carbon arc. The photographic intensities in the carbon arc spectrum are determined at the wave-lengths of the lines in the spectrum under examination by the use of equation (2). Since the temperature of the carbon arc is the temperature of boiling carbon, it does not vary with variations in the current. The temperature correct to within a few degrees is 3750° C. abs. On the assumption that the positive pole acts as a black body, an assumption closely justified since the crater is an enclosure, the absolute distribution of energy in the spectrum may be calculated with sufficient accuracy by Wien's law,

$$I_\lambda = c\lambda^{-5} e^{-a/\lambda\tau},$$

where $a = 1{\cdot}445$ cm. deg., τ is the temperature, and c is a constant which cancels out when the intensities are computed relative to the intensity of a given wave-length taken as unity. The relative intensities on an absolute scale in the spectrum under examination are then given by the formula

$$\alpha\beta/\gamma \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where α is the photographic intensity at λ in the spectrum under examination, β is the absolute intensity of the carbon arc at λ, and γ is the photographic intensity of the carbon arc at λ.

If a prism spectroscope be used, the dispersion is not normal and, according to the nature of the spectrum under examination, it may be necessary to make a correction in the above formula for dispersion. If the spectrum is continuous, both it and the carbon arc are alike affected by the inequality in dispersion, and no correction is needed. If the unknown spectrum be a line spectrum, then inequality in dispersion does not affect the central intensities of the lines, but it does change the apparent intensity distribution in the carbon arc. If $I_\lambda d\lambda$ be the intensity computed from Wien's formula, then the range of wave-lengths $d\lambda$ falling on a given portion of the photographic plate, dx, varies with λ. It is accordingly necessary to multiply the absolute intensities by the factor $\frac{d\lambda}{dx}$. The dispersion of a prism is satisfactorily given by Hartmann's formula,

$$\lambda = \lambda_0 + \frac{c}{x - x_0} \quad \text{and} \quad \frac{d\lambda}{dx} = \frac{c}{(x - x_0)^2} = \frac{(\lambda - \lambda_0)^2}{c}.$$

These dispersion factors are computed relative to some wave-length arbitrarily chosen as unity. Thus formula (3) will give the intensities relative to some wave-length on an absolute scale.

2. *Experimental Details.*

The wedge, supplied by Hilger, is of approximate dimensions $6 \times 4 \times 1$ mm. It is made of very dense neutral tint glass transmitting less than 30 per cent. of the light at the thin end. The ratio of densities thin : thick is $\frac{1}{21}$. The constants ρ_λ and D_λ were determined in the following manner. The wedge was mounted in front of the slit of a Brashear laboratory spectroscope, and an exposure of the iron arc spectrum was made. On an adjacent portion of the same plate a second exposure of equal duration was made with the intensity of the arc reduced by $\frac{1}{10}$. The intensity was reduced by the use of a sector with a 36° opening making less than 100 R.P.M. (to avoid the intermittency error). The iron arc was mounted according to the method suggested by Pfund, but did not burn absolutely steadily. Accordingly, four pairs of exposures were made. From the theory in the preceding section it may readily be shown that $\rho_\lambda = \frac{H}{l \tan a (h_\lambda'' - h_\lambda')}$ and $D_\lambda = \frac{H}{h_\lambda'' - h_\lambda'}$, where h_λ'', h_λ' are the heights respectively of a given line with the normal intensity and the intensity reduced by $\frac{1}{10}$. In order to determine the values of l, a, δ (see fig. 1), thirteen pairs of measurements of the wedge were made on the measuring engine. A solution by least squares gave the following values :—$l = 6{\cdot}436$ mm. ; $\tan a = 0{\cdot}1266$; $\delta = 0{\cdot}239$ mm. Negative enlargements were made in the way already described and the heights h_λ'', h_λ' for a number of lines were determined. The mean values for the four pairs of exposures were taken and smoothed by a curve. The necessary values are given in Table I.

TABLE I.

λ.	D_λ.	$\rho_\gamma\delta$.	λ.	D_λ.	$\rho_\lambda\delta$.
0·42μ	4·8	1·41	0·48μ	3·9	1·15
0·43	4·5	1·34	0·49	3·8	1·12
0·44	4·4	1·27	0·50	3·8	1·10
0·45	4·2	1·24	0·51	3·7	1·10
0·46	4·1	1·19	0·65	3·6	1·05
0·47	4·0	1·17			

The single prism spectrograph used in this work is mounted at the focus of the Cassegrain combination of the 72-in. reflector—aperture ratio $\frac{1}{18}$. In order to reduce the exposure time to a minimum the short focus camera (less than $\frac{1}{3}$ focal length of the collimator) was employed with a linear dispersion of 50 Å. per mm. at Hγ. For the same reason the slit was opened to a width of 0·074 mm. giving a purity of 5000. The wedge was mounted in a small holder which slipped over the slit. Only the thin end of the wedge was used, the thicker half being occulted by black paper. This also helped to reduce the exposure time. The method adopted to secure uniform illumination on the slit was

to trail the star backward and forward. This was readily accomplished by the slow motion in R.A., which alters the rate of the driving clock. The star was observed and kept on the slit by means of a guiding telescope. On account of the denseness of the wedge and in spite of the steps taken to shorten the exposure, 90 minutes was the minimum exposure for the fast Seed 30 plates used in the blue region. As the time of trail was approximately 70 seconds, the star passed over the undiaphragmed length of the wedge at least 80 times per exposure. In this way uniformity of illumination was secured.

Since the reflectivity of the silver coats on the mirrors is a function of λ and also varies with the time, it was found necessary to mount the carbon arc in such a way that the optical paths of the light from the star and the arc were as far as possible identical. The arc was mounted at the upper end of the telescope tube, and an image of its positive crater was focussed on a pin-hole by a condensing lens. This pin-hole was at the focus of a 3-in. Brashear objective which directed the light in a parallel beam on to the main mirror. Thence its optical path was identical with that of the star, save for the interposition of a ground glass screen to secure uniform illumination of the slit and collimator. The part of the optical path peculiar to the star was the earth's atmosphere, and the part peculiar to the arc was the condensing system of two lenses and the ground glass screen. The effect of the condensing system and ground glass screen can readily be eliminated by two spectra of the carbon arc taken in the laboratory, the one with and the other without this system.

Preliminary work was commenced early in December. Owing to a prolonged spell of cloudy weather, it was not until the middle of February that any satisfactory spectra could be obtained. By this time γ Cassiopeiæ was three hours west at sunset and the maximum possible exposure was two hours.

Three satisfactory spectra were secured on Seed 30 plates, and one weak spectrum showing only H_α and H_β was obtained on an Ilford panchromatic plate. Fig. 2 is a negative enlargement of a Seed 30 plate secured on Feb. 28, and shows the star spectrum and the carbon arc spectrum (with iron arc comparison) photographed through the wedge. Negative enlargements were made of these four plates through a process screen with 100 lines to the inch. The measurement and reduction were carried out in the manner previously indicated.

3. *The Intensity Distribution in the Continuous Spectrum.*

In order to show in detail the computation, the complete reduction of the Seed 30 plate taken on February 19 is given in Tables II. and III. Table II. is self-explanatory as the reduction closely follows equation (2) for the photographic intensity, viz.

$$I_\lambda/I_c = \log_{10}{}^{-1}(D_\lambda h_\lambda/H + \rho_\lambda \delta).$$

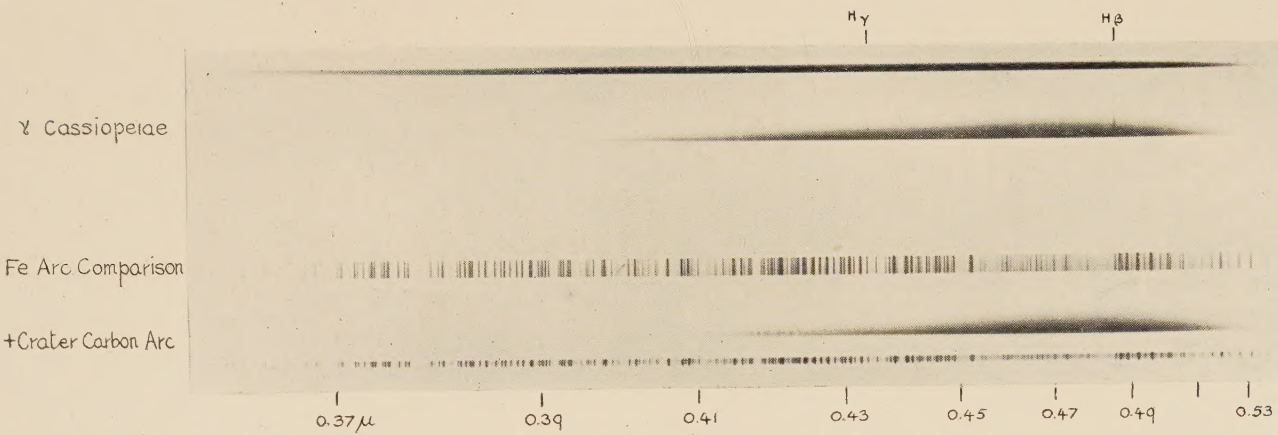

Fig. 2.—Wedge Spectrum of γ Cassiopeiæ and the Carbon Arc.

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The value H = 1·37 and is constant throughout the table. The seventh column contains I_λ/I_c as computed from the formula, and

TABLE II.

Photographic Intensities in Star Spectrum, Feb. 19.

λ.	h_λ.	D_λ.	$D_\lambda h_\lambda/H$.	$\rho_\lambda\delta$.	$\log I_\lambda/I_c$.	I_λ/I_c.	$I_\lambda/I_{0\cdot45}$
0·43μ	0·10	4·5	0·33	1·34	1·67	46·8	9·34
0·44	0·14	4·4	0·43	1·27	1·70	50·1	10·00
0·45	0·15	4·2	0·46	1·24	1·70	50·1	10·00
0·46	0·19	4·1	0·56	1·19	1·75	56·2	11·21
0·47	0·18	4·0	0·52	1·17	1·69	49·0	9·78
0·48	0·18	3·9	0·51	1·15	1·66	45·7	9·12
0·49	0·12	3·8	0·33	1·12	1·45	28·2	5·63

the final column contains the intensities expressed as ratios of the intensity 0·45μ arbitrarily taken as 10·00.

Table III. contains the reduction of the photographic intensities to absolute intensities and the correction for atmospheric

TABLE III.

Absolute Intensities in Star Spectrum, Feb. 19.

λ.	Phot. Int. Star = α.	Abs. Int. Arc = β.	Phot. Int. Arc = γ.	Abs. Int. Star = $\frac{\alpha\beta}{\gamma}$	$\alpha \frac{\sec z}{\lambda}$	I_0.
0·43μ	9·34	8·33	4·95	15·72	0·958	16·41
0·44	10·00	9·25	6·98	13·26	0·978	13·55
0·45	10·00	10·00	10·00	10·00	1·000	10·00
0·46	11·21	10·73	12·95	9·28	1·019	9·11
0·47	9·78	11·65	17·37	6·56	1·036	6·33
0·48	9·12	12·43	14·89	7·61	1·053	7·23
0·49	5·63	13·15	9·09	8·15	1·067	7·64

absorption. In the second column are the photographic intensities brought down from the final column of the preceding table. The third column contains the absolute intensities of the carbon arc as computed from Wien's formula, and expressed as ratios of the intensity at 0·45μ taken as 10·00. No correction is necessary for dispersion, since both the star and the carbon arc spectrum are alike affected by the inequality in dispersion. In the fourth column are the photographic intensities of the carbon arc. The correction to these values, to remove the condensing system of two lenses and the ground glass screen from the optical path, proved on experiment to be less than the accidental errors of measurement. Accordingly no corrections for this cause are applied to the photographic intensities of the carbon arc. The fifth column contains the absolute intensities as computed from the formula $\alpha\beta/\gamma$, the intensities being expressed as ratios of the intensity at 0·45μ taken arbitrarily as 10·00.

In order to correct the intensities as thus computed for atmospheric transmission, Bouguer's formula is used, viz.

$$I_m = I_0 a_\lambda^{\sec z},$$

where I_m is the observed intensity, I_0 is the intensity outside the earth's atmosphere, a_λ is the transmission coefficient for the wave-length λ, and z is the zenith distance. The most satisfactory method to determine a_λ and I_0 is to obtain the intensity distribution in the star spectrum at two zenith distances. This gives a complete solution of the equation for each wave-length. On account of the long exposure required (viz. 2 hours) this procedure was impossible. Accordingly the values of $a_\lambda^{\sec z}$ were computed from the tables given by Fowle,* assuming a vapour pressure of 0·50 cms. and an elevation above sea-level of 221 metres. The values for Feb. 19 are given in the sixth column of Table III. It may be noted that the transmission coefficients for the later dates differ from those of Feb. 19 by at the most only one figure in the second place. The last column of Table III. contains the absolute intensities as they would appear outside the earth's atmosphere.

The intensities for the two other Seed 30 plates were computed in a similar manner, and the results are summarised in Table IV.

TABLE IV.

λ.	Feb. 19.		Feb. 24.		Feb. 28.		Mean I_0.	Mean Error %
	Phot. Int.	I_0.	Phot. Int.	I_0.	Phot. Int.	I_0.		
0·42μ			7·57	14·47	7·09	13·57	14·02	3·21
0·43	9·34	16·41	10·00	17·59	8·30	14·60	16·20	6·61
0·44	10·00	13·55	10·64	14·43	10·49	14·23	14·07	2·49
0·45	10·00	10·00	10·00	10·00	10·00	10·00	10·00	
0·46	11·21	9·11	10·45	8·49	10·21	8·29	8·65	4·05
0·47	9·78	6·33	10·45	6·76	12·05	7·79	6·96	7·90
0·48	9·12	7·23	8·89	7·05	8·51	6·73	7·00	2·57
0·49	5·63	7·64	5·11	6·93	5·63	7·62	7·40	4·19
0·50			3·23	6·47	3·64	7·27	6·87	5·82
0·51					1·66	6·06		

The photographic intensity and the absolute intensity corrected for atmospheric transmission are given for each plate. The absolute corrected intensities are also shown graphically in fig. 3, where the open circles are the intensities for the plates taken on Feb. 19, the barred circles the values for Feb. 24, and the black circles the values for Feb. 28.

To determine the consistency of the results, the absolute corrected intensities I_0 in Table IV. are meaned and the result is given in the eighth column. The residuals from this mean for each plate and each λ were computed and their arithmetic mean

* *Ap. J.*, 38, 392.

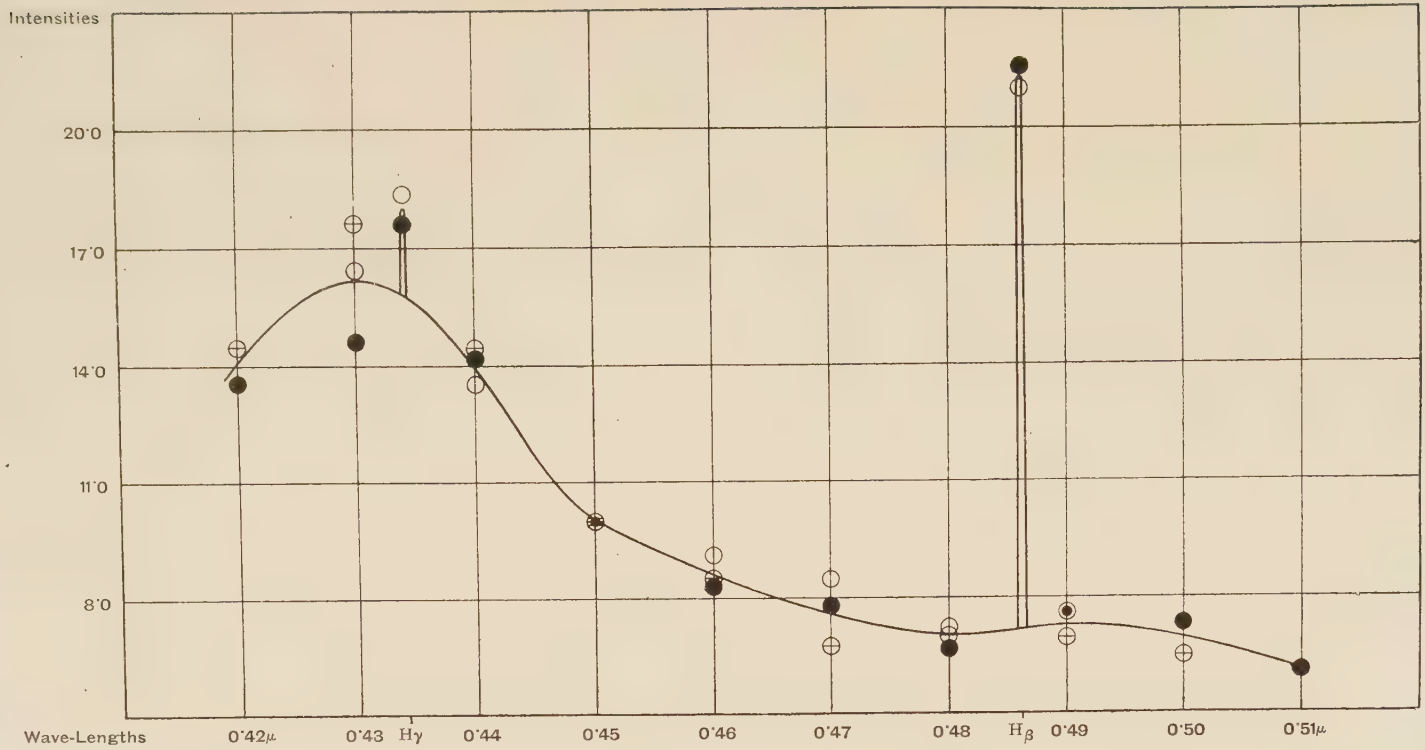

Fig. 3.—Distribution of Energy in the Spectrum of γ Cassiopeiæ.

taken. In the last column of the table this mean error is expressed as a percentage of the mean absolute intensity in the preceding column. It will be observed that the mean of these errors is 4·6 per cent., or, in other words, the final results are accurate within 5 per cent. This error compares not unfavourably with the 1 per cent. accuracy obtained by Merton and Nicholson under laboratory conditions. In future work with a less dense wedge, now on order, of which the full height can be used, it is hoped to secure a higher degree of accuracy.

Referring back to fig. 3, it will be noted that there is a maximum of intensity at 0·43μ. From Wien's displacement formula,

$$\lambda_{max.}\, T = 2930,$$

on the assumption that the photosphere radiates as a black body, the effective temperature of γ Cassiopeiæ is 6800° A. This is a much lower temperature than commonly accepted for B type stars. The approximate values obtained by previous investigators are :—

Wilsing and Scheiner . . .	9600°A.
Nordmann	18,000 – 40,000°A.
Rosenberg	50,000°A.
Coblentz	4700 – 6000°A.

Coblentz's temperatures were obtained from radiometric measures of the total energy of certain of the brighter stars. Measures were made with and without the absorption of a water cell. From Planck's law he was able to deduce the temperatures with an accuracy of 200°. The previous work was done in the visual region with some form of spectrophotometer.

It is also possible, under certain simplifying assumptions, to derive the temperature of the early B type stars from the presence in their spectrum of the super-spark helium line 4686. All evidence seems to indicate, that the primary condition for the production of a line spectrum by an element is the ionisation of its atoms, the second condition being of course the return of an electron to the atomic system. On the assumption that there is no electrostatic field in the photosphere of the star, the only factor that will produce ionisation is the photo-electric effect. The energy of collision under any reasonable assumptions of temperature is not sufficient. From Bohr's theory the frequency required to ionise an atom with one electron is given by $\frac{2\pi^2 N m e^4}{h^3}$, where N is the atomic number of the element, m and e the mass and charge respectively of the electron, and h is Planck's constant. The hydrogen atom when emitting the Balmer series, and the helium atom when emitting the 4686 and the Pickering series, have only one electron. Inserting the proper values in the above formula, it is found that radiation of wave-length 911·7 Å. will ionise the hydrogen atom, and radiation of wave-length 227·8 Å.

will ionise the helium atom. Now the Balmer series appears in M type stars at a commonly accepted temperature of 3000°A. On the assumption that stars radiate as black bodies, from Planck's law,

$$I_\lambda = c_\lambda^{-5}/(e^{a/\lambda\tau}-1),$$

it is possible to compute the energy of wave-length 911·7 Å. for a temperature of 3000°A. It is 63·8 ergs. As a first approximation it may be assumed that, if 63·8 ergs of radiation 911·7 Å. ionise enough hydrogen atoms to produce a visible spectrum, then 63·8 ergs of radiation 227·8 Å. will ionise enough helium atoms to produce the line 4686. Inserting the necessary values in Planck's formula, it is possible to compute at what temperature the radiation 227·8 Å. will have an energy of 63·8 ergs. The temperature as thus found is 10,600°A. On the assumption, then, of black body radiation and eliminating unknown electrostatic forces, the effective temperature of the early B type stars may be taken to a first approximation as about 10,000°A.

Three possible hypotheses may be offered in explanation of the observed energy distribution. The evidence is not yet sufficient to suggest the correct explanation.

(1) The maximum is not at 0·43μ. The placing of the maximum at 0·43μ really depends upon the two observations at 0·42μ. As these were just at the limit of accurate measurement it is possible that there is an error.

(2) The true maximum is obscured by scattering in the upper layers of the star's atmosphere. A terrestrial analogue of this is the blue colour of the sky. Since the scattering varies inversely as the fourth power of the wave-length, the maximum of intensity in sky light should be in the infinitely short waves. However, the lower layers of the atmosphere scatter this sky radiation, so that finally, as received on the earth, the observed maximum is in the blue. Something of the same nature may take place on the star.

(3) The photosphere does not radiate as a black body. Hence estimates of temperature based on Wien's and Planck's laws will give erroneous results. This would in some measure account for the widely discrepant results obtained by different investigators. The existence of a secondary maxima at 0·49μ (fig. 3) may be taken as evidence in favour of this hypothesis. The radiation of black bodies shows only one maximum in the energy curve.

4. *The Intensity Decrement in the Balmer Series.*

In Table V. are given the photographic intensities and the absolute intensities corrected for atmospheric transmission of the hydrogen lines. The values for Hα, Hβ are from the Ilford panchromatic plate obtained on Feb. 21, and the values Hβ, Hγ are the means from the Seed 30 plates taken on Feb. 19 and Feb. 28. They are combined on the assumption that there

was no variation in line intensity on these three dates. The reduction was performed in the same manner as before with the addition of a correction for dispersion. In fig. 3 the absolute intensities of Hβ and Hγ relative to the continuous spectrum are shown graphically.

TABLE V.

Intensities of the Hydrogen Lines.

Line.	γ Cassiopeiæ.		Laboratory.	
	Phot. Int.	Abs. Int.	Hydrogen.	Hydrogen+Neon.
H_α	58·20	10·00	10·00	10·00
H_β	29·00	3·95	2·61	3·38
H_γ	14·42	3·33	1·86	3·47

For comparison, in the remaining columns of the table, are given the intensities of the Balmer series as obtained by Merton and Nicholson. To facilitate comparison the star and laboratory values are all reduced to the value Hα = 10·00. As is shown in the particular values quoted, and as has been confirmed in a later paper,* the intensity in a series goes into the higher numbers when there is a mixture of gases in the discharge tube. The intensity decrement of the Balmer series in the star is of the same order as that shown by hydrogen and neon in the laboratory.

The matter is complicated by some evidence of variation in the intensities obtained on the three Seed 30 plates. The absolute intensities relative to 0·45μ taken arbitrarily as 10·00 are given in Table VI.

TABLE VI.

Line.	Feb. 19.	Feb. 24,	Feb. 28.
Hβ	21·08	17·88	21·59
Hγ	18·41	18·31	17·59

It will be noted that there is a decided change in the intensity decrement on Feb. 24 which is well outside the range of accidental error. Variations in the intensity of the emission lines in γ Cassiopeiæ have been noted before. This confirms that variation and suggests a cause, namely, a varying amount of some other gas in the midst of the vibrating hydrogen atoms.

In some work on the intensity distribution in emission lines, Merton and Nicholson have shown that for ordinary discharges the intensity distribution is given by $I = I_0 e^{-k^2x^2}$, and that for condensed discharges it is given by $I = I_0 e^{-kx}$. When the emission lines are photographed through a wedge, these distributions correspond to a parabolic contour in the case of the ordinary discharge, and a straight line contour in the case of a condensed discharge. It is thus possible from the inspection of a wedge photograph to determine the nature of the discharge. Fig. 2

* *Roy. Soc. Proc.*, 96, 112.

shows that the contour of Hβ is a parabola. Another spectrum taken with a narrower slit and a linear dispersion of 30 A. to the mm. confirms this. There is thus little doubt that the hydrogen emission lines in γ Cassiopeiæ are excited by an ordinary discharge. This is peculiar in view of the number of lines in the spectrum, such as 4686, which require highly condensed discharges to produce them. The suggestion is perhaps that the hydrogen emission lines are produced at higher levels under different conditions.

5. *Conclusions.*

Apart from the results obtained, the paper shows the astronomical value of Merton and Nicholson's method of determining spectral intensities. It makes possible a knowledge of the actual intensity distribution in the visible spectrum of the brighter stars. Applied to stars showing emission lines, it should give an insight into the conditions of their production, and, with sufficient dispersion, a knowledge of the intensity distribution in the lines themselves. In this connection the possibility of detecting a rotational effect should be borne in mind. With a less dense wedge, now on order, it is proposed to carry out some of these investigations.

The writer wishes to acknowledge the valuable suggestions made by the Director throughout the course of this investigation, as well as his permission to undertake it.

Dominion Astrophysical Observatory,
Victoria, B.C.
1920 *May* 8.

On the Motion of the Magellanic Clouds. By Ejnar Hertzsprung.

(*Communicated by the Astronomer Royal.*)

In the *Publications of the Lick Observatory*, vol. xiii., pt. 5, R. E. Wilson discusses the radial velocities of 17 nebulæ in the greater Magellanic Cloud. The mean radial velocity is + 276 km./s., but there are marked differences depending on the co-ordinates of the objects —the greater the south polar distance, the more positive the radial velocity is found to be. As an explanation of this fact, Wilson suggests rotation of the Cloud, but states "that the evidence for this is at present so meagre as to assign the hypothesis to the domain of speculation."

However, no particular suggestion is needed to explain the differences in radial velocity mentioned—the simplest assumption, viz. that all parts of the Cloud are moving parallel and with the same speed in space, will do. And more than that: The radial velocity of + 168 km./s. shown by the only nebula measured in the lesser Magellanic Cloud is consistent with the supposition that the two Clouds share the same motion. The following table will

show the validity of this statement. The radial velocities have been calculated from the formula

$$V = -59{\cdot}5 \sin\delta + 228{\cdot}5 \cos\delta \cos\alpha + 578{\cdot}5 \cos\delta \sin\alpha,$$

which has been derived according to least squares, assigning equal weight to the radial velocities of all 18 nebulæ. Calling $\sin\delta = x$, $\cos\delta\cos\alpha = y$ and $\cos\delta\sin\alpha = z$, the mean error of the computed radial velocity in the direction α, δ is

$$\pm 12{\cdot}5 \text{ km./s.} (5{\cdot}3\,x^2 + 38{\cdot}2\,y^2 + 26{\cdot}5\,z^2 + 23{\cdot}5\,xy + 23{\cdot}2\,xz + 47{\cdot}0\,yz)^{\frac{1}{2}}.$$

The total motion is found to be 625 km./s. ± 80 km./s. (m.e.) towards $4^h\,34^m \pm 21^m$, $-5^\circ{\cdot}5 \pm 3^\circ{\cdot}3$, or corrected for the sun's motion 608 km./s. towards $4^h\,31^m$, $-4^\circ{\cdot}7$ (galactic co-ordinates: $l = 168^\circ$, $b = -30^\circ$).

According to the differences O − C, the mean error of a radial velocity is ± 12·5 km./s., while we should expect ± 11 km./s., judging from the deviations between the single measures of the same nebula together with the number of observations of each nebula. This means that the radial velocities found are in accordance with the supposition of common motion in space of all the objects observed. As no sensible internal motion occurs, there is hope that more accurate radial velocities will improve our knowledge as to the motion of the system as a whole.

It may be added that the radial velocities of the 17 nebulæ in the greater Magellanic Cloud taken alone yield the formula

$$V = -5 \sin\delta + 172 \cos\delta \cos\alpha + 758 \cos\delta \sin\alpha.$$

This formula gives for the place of the lesser Cloud an expected radial velocity of + 102 km./s. ± 54 km./s., while the observed value is + 168 km./s. Hence the radial velocity of the lesser Cloud is not more different from that of the greater Cloud, + 276 km./s., than we should expect from the radial velocities observed in different parts of the greater Cloud. This is in accordance with the small deviation O − C = + 3 km./s. shown by the only object of the lesser Cloud in the general table of radial velocities.

The question arises, what proper motions of the Magellanic Clouds we might expect according to the above result. The angular distance of the greater Cloud ($5^h\,18^m$, $-68^\circ{\cdot}7$) from the point of convergence ($4^h\,34^m$, $-5^\circ{\cdot}5$) is found to be $63^\circ{\cdot}7$, and of the lesser Cloud $72^\circ{\cdot}2$. The velocity at right angles to the line of sight is then 560 km./s. ± 89 km./s. (m.e.) for the greater Cloud and 603 km./s. ± 73 km./s. for the lesser one, the mean error being 15 per cent. in the first and 12 per cent. in the latter case. Assuming a parallax of $''{\cdot}0001$ (*Astron. Nachr.*, 4692, 196, 204, 1913), we have to expect angular proper motions of $''{\cdot}0118$ and $''{\cdot}0127$ annually for the two Clouds respectively. This is a fair amount, which it should be possible to verify in less than a century, and thus enable us to determine the parallax.

The direction of the proper motion is 348° for the greater and 55° for the lesser Cloud. Accordingly the two Clouds, which are separated by 20°, approach each other $''{\cdot}0018$ annually.

Table of Radial Velocities.

α (1900).	δ (1900).	O. km./s.	Number of Plates.	C. km./s.	O−C. km./s.
h m	° ′				
1 6·2	−73 44	+168	4	+165	+ 3
4 49·9	−69 21	+293	1	+275	+18
52·0	−67 6	+309	3	+296	+13
52·6	−69 33	+265	4	+272	− 7
54·6	−69 21	+260	4	+274	−14
55·1	−69 21	+256	3	+274	−18
56·6	−66 34	+296	2	+301	− 5
4 56·8	−66 32	+292	3	+301	− 9
5 22·2	−68 3	+299	3	+283	+16
22·5	−68 4	+292	3	+282	+10
25·6	−68 33	+260	1	+277	−17
35·6	−67 39	+297	3	+283	+14
39·4	−69 9	+275	4	+268	+ 7
40·5	−69 42	+259	3	+263	− 4
40·6	−69 49	+255	4	+261	− 6
40·6	−69 42	+251	5	+263	−12
41·0	−69 43	+264	1	+262	+ 2
5 41·4	−69 43	+271	5	+262	+ 9

Leiden:
1920 *August* 6.

Photographic Positions of Neptune obtained between March and May 1920. By W. J. Luyten.

In order to obtain exact positions of Neptune during the time it was not possible to observe it with the transit circle, a series of photographic observations was begun on 1920 March 18 and continued till Neptune disappeared in the twilight, thirteen plates having been obtained. The instrument used was the photographic refractor of Leiden Observatory (aperture 33 cm., scale 39″·3 per mm.), which Professor Hertzsprung kindly placed at my disposal for this purpose.

As Neptune was near its stationary point, and during the whole period of observation moved only 15′, it was possible to simplify reduction by keeping to one plate centre.

Exposures were given, ranging from one to three minutes, on Ilford Monarch plates, on which a réseau was printed in the usual way. By the kind permission of the Astronomer Royal, the plates

were measured in the Greenwich Astrographic micrometer, during my stay at the Royal Observatory. A saving in the labour of reduction was effected by choosing one plate as standard and reducing the remaining plates to it by means of comparison stars; for in this case the corrections for scale are quite small, although not negligible on account of refraction. Then these comparison stars were reduced to the standard plate by means of the constants derived from themselves, and the mean of the results taken. The deviations from the mean yielded a mean error of $0''\cdot17$ in R.A. and $0''\cdot22$ in Dec.

The standard plate, now with thirteen positions of Neptune, was then reduced to R.A. and Dec. by means of all the A.G.C. stars (zone Berlin A) occurring on it, viz. Nos. 3526, 29, 33, 34, 35, 41, 44, 47, and 52; the two stars 3536 and 3543 being excluded in the ultimate reduction, as they showed too large proper motion. The results thus obtained and corrected for apparent place and parallax are given in Table I. below, for which no further explanation is necessary. To compare these places of Neptune with the ephemeris in the N.A., a correction according to the different fundamental system employed had to be applied. This was done by means of the tables given in Boss's P.G.C., and the difference N.A.–A.G.Be A found to be $-0^s\cdot024$ and $+0''\cdot49$; the differences Leiden–N.A. thus obtained are given in Table I., columns 4 and 6. (No correction for mean parallactic motion of the reference stars since the epoch of the catalogue has been applied.)

TABLE I.

Date 1920.	G.M.T.	App. R.A.	Corr. to N.A.	App. Dec.	Corr. to N.A.
	h m	h m s	s	° ′ ″	″
Mar. 18	11 2	8 45 46·290	−0·11	+17 59 47·46	+2·1
22	8 0	31·766	·08	18 0 46·69	1·9
25	7 59	21·636	·11	1 28·14	1·8
29	10 30	9·604	·07	2 17·65	1·6
Apr. 1	9 0	2·260	·08	2 49·06	2·3
16	9 29	44 42·038	·07	4 17·83	2·1
21	9 53	41·985	·06	4 21·78	2·3
23	9 38	42·909	·07	4 19·13	1·8
May 4	8 52	57·572	·09	3 29·82	1·9
9	10 21	45 9·686	·07	2 47·29	2·7
11	9 49	15·306	·09	2 26·25	2·1
14	9 34	24·808	·06	1 50·95	2·0
18	9 58	39·284	·06	0 56·87	1·5
			$-0^s\cdot078\pm\cdot005$		$+2''\cdot08\pm\cdot09$

For the two stars Nos. 3536 and 3543 new places were derived from the plates, which are given below, together with the difference from the A.G.C. places reduced to 1920·0.

Star.	R.A. 1920·0.	Leiden – A.G.	Dec. 1920·0.	Leiden – A.G.
	h m s	s	° ′ ″	″
3536	8 45 30·211	−0·12	+18 8 42·58	−3·9
3543	8 46 6·474	−0·53	+17 59 21·86	−2·9

Royal Observatory, Greenwich:
1920 *September* 9.

Orbit of Holden 60. By J. Jackson, M.A., B.Sc.

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This pair is No. 12696 of Burnham's General Catalogue, where the magnitudes of the two components are given as 8·5, 8·9. The star is B.D. +38°, 5112, and the position for 1925 is R.A. $23^h\ 57^m\ 33^s$, Dec. +39° 12′.

The orbit was investigated by Fox in *Pop. Ast.*, 17, 17, using observations to 1909. This was considered to be a more or less uncertain approximation, and in the *Annals of the Dearborn Observatory*, 1, 43, Fox states that it is now (1914) evident that the period will be much longer than the forty years originally found, and that it may well be two or three times as great. I have recomputed the orbit, using Fox's observations to 1914, and while I find the period nearly the same as that originally found by Fox, the other elements do not agree very well. It appears to be impossible to fit satisfactorily both of the single observations of 1893 and 1902. The uncertainty of the elements is largely the result of the lack of observations between these dates. The orbit obtained does not suit the latest observations so well as is desirable, and the period may be somewhat longer than that given, although it should be less than fifty years.

The elements obtained are (for notation see *M.N.*, 80, 544):—

T	1915·42	a	″0·50	i	69°·7
P	40·76 years	e	0·35	Ω	119°·1
n	+8°·83			ω	114°·2

These elements give the following ephemeris:—

1920·0	134°·2	″0·332
1922·0	124·7	0·448
1924·0	118·6	0·518
1926·0	113·6	0·547
1928·0	108·9	0·544
1930·0	103·7	0·518

The following is the comparison of observed with computed positions:—

	Obs.	Comp.	O−C.	Obs.	Comp.	O−C.	
	°	°	°	″	″	″	
1881·71	124·1	123·1	+ 1·0	0·62	0·47	+·15	β 3
1888·17	105·5	106·5	− 1·0	0·49	0·54	−·05	Com. 3
1893·13	91·1	90·9	+ 0·2	0·25	0·41	−·16	Com. 1
1902·62	345·5	356·2	−10·7	0·45	0·25	+·20	β 1
1905·68	331·4	327·6	+ 3·8	0·28	0·34	−·06	β 2
1906·72	326·7	322·2	+ 4·5	0·23	0·36	−·13	β 1
1907·69	316·5	317·2	− 0·7	0·31	0·38	−·07	β 1
1908·03	297·9	315·4	−17·5	0·65	0·39	+·26	Gr. Obs. 1
1909·22	306·3	310·0	− 3·7	0·26	0·40	−·14	Fox 2
1910·78	301·9	303·0	− 1·1	0·26	0·40	−·14	Doo. 2
1911·59	295·4	299·4	− 4·0	0·29	0·38	−·09	Fox 5
1912·59	289·6	294·1	− 4·5	0·38	0·35	+·03	Fox 3
1913·71	290·2	286·3	+ 3·9	0·38	0·29	+·09	Fox 3
1914·64	281·7	276·2	+ 5·5	0·39	0·23	+·16	Fox 1

The hypothetical parallax is ″·0335.

Elimination of the known Integrals from the n-Body Problem.

By Professor L. Becker, Ph.D.

1. In the n-body-problem the $3n$ independent co-ordinates are referred to a fixed system. Six integrals belong to the motion of the centroid and can be eliminated by the introduction of $3(n-1)$ Jacobi co-ordinates, in which the system of axes moves with the centroid parallel to itself. Other three integrals give the position of the invariable plane and the angular momentum of the system. In this paper I shall introduce $3(n-1)$ new co-ordinates, and as many quantities which are canonically conjugated to them, and they are so chosen that four of them disappear from the characteristic function. Three of these four quantities have constant values, and express the same result as is contained in the three integrals mentioned. The latter are thus eliminated from the problem, and the problem is reduced to the order $6(n-1)-4=(6n-10)$. The problem has been solved before, though for different co-ordinates and by different methods.*

In my method I use for the relative position of the masses the plane passing through the three masses m_1, m_2, m_3 as plane of reference, with the line m_1m_2 as zero line, and I employ the three

* As to references, see E. T. Whittaker, "Prinzipien der Stoerungstheorie . . ." in the *Encyklopaedie der mathematischen Wissenschaften*, 6, 2, 12, pp. 516, 521.

Eulerian angles for defining the position of the reference system. I finally eliminate Euler's angles and their conjugated quantities by the same contact transformation as is applied in the problem of rotation.

2. *Characteristic Function expressed in Relative Polar Co-ordinates and Euler's Angles.*—Let X, Y, Z designate Jacobi's co-ordinates, m_2 being referred to m_1, m_3 to the centroid, G_1, of

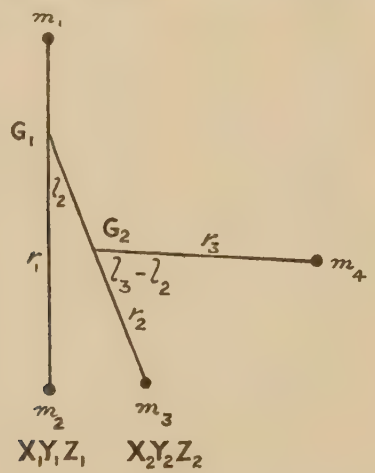

FIG. 1.

m_1 and m_2, m_4 to the centroid of m_1, m_2, m_3, etc. (see fig. 1). The kinetic energy of the system then is given by

$$T = \sum_1^{n-1} \frac{\mu_k}{2} [\dot{X}_k^2 + \dot{Y}_k^2 + \dot{Z}_k^2],$$

where μ_k are the well-known functions of the masses.

I introduce Euler's angles ψ, θ, ϕ, and the relative polar co-ordinates, viz. the distance, r_k; the longitude, l_k; and latitude, b_k. They are defined in section 1 and are shown in figs. 1 and 2. The total number of variables is again $3(n-1)$, since l_1, b_1, and b_2 have zero values by definition.

$$X = r\{\cos\psi \cos(\phi + l) \cos b + \sin\psi \cos\theta \sin(\phi + l) \cos b + \sin\psi \sin\theta \sin b\}$$

. . .

T becomes

$$T(r\dot{r} \ldots) = \sum_1^{n-1} \frac{\mu}{2} \{\dot{r}^2 + r^2[(\dot{\phi} + \dot{l}) \cos b - \dot{\psi}(\cos\theta \cos b + \sin\theta \sin b \sin\phi + \dot{\theta} \cos(\phi + l) \sin + r^2[\dot{\psi} \cos(\phi + l) \sin\theta + \dot{\theta} \sin(\phi + l) - \dot{b}]^2\}.$$

The suffixes, k, have been omitted.

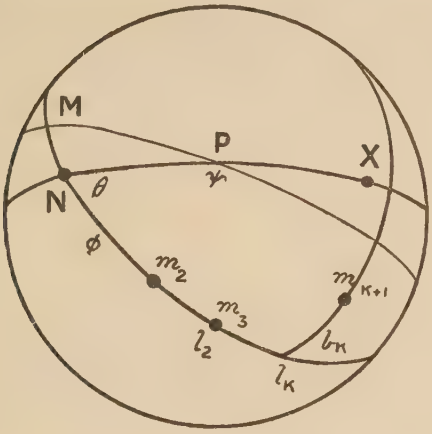

FIG. 2.

Use capital letters to denote the quantities which are canonically conjugated to the co-ordinates, *e.g.* $L_k = \partial T(r\dot{r} \ldots)/\partial \dot{l}_k$, then the characteristic function, $H = T - U$, becomes

$$(R, r \ldots) = \frac{1}{2\mu_1}\left\{ R_1^2 + r_1^{-2}\left(\Phi - L_2 - \sum_3^{n-1} L\right)^2 + (r_1 \sin l_2)^{-2}\left[F_1 + \sum_3^{n-1}(L \tan b \cos(l - l_2) - B \sin(l - l_2))\right]^2 \right\}$$

$$(1) \quad + \frac{1}{2\mu_2}\left\{ R_2^2 + r_2^{-2} L_2^2 + (r_2 \sin l_2)^{-2}\left[F_2 + \sum_3^{n-1}(L \tan b \cos l - B \sin l)\right]^2 \right\}$$

$$+ \sum_3^{n-1} \frac{1}{2\mu}\left[R^2 + (r \cos b)^{-2} L^2 + r^{-2} B^2\right] - f \sum \frac{m_a m_b}{r_{ab}}$$

where

$$F_1 = -\Theta \cos(\phi + l_2) + (\Psi + \Phi \cos\theta) \sin(\phi + l_2)/\sin\theta,$$
$$F_2 = -\Theta \cos\phi + (\Psi + \Phi \cos\theta) \sin\phi/\sin\theta.$$

ψ and the time do not occur in H, and hence H and Ψ have constant values.

3. *Contact Transformation.*—The terms F_1 and F_2 are analogous to those in the characteristic function in the problem of rotation. As already mentioned, I make the same change of variables. Instead of the three Eulerian angles, ψ, θ, ϕ, I introduce the arcs shown in heavy lines in fig. 3.

According to the diagram the three independent variables can be replaced by not less than five quantities, *i.e.* the three arcs m_2M,

MP, PX, and the two angles at M and P. It can be proved that if the three arcs be chosen as the new co-ordinates (q_1, q_2, q_3), the angles at M and P are functions of the quantities "p" which are canonically conjugated to them. I apply the following transformation:

$$q_1 = \psi - a, \qquad \Psi = p_1$$
$$q_2 = q_2\,(\theta, \mathrm{A}, \mathrm{B}), \qquad \Theta = -p_2 \sin b \sin \mathrm{A}$$
$$q_3 = \phi + b, \qquad \Phi = p_3$$
$$\cos \mathrm{A} = -\frac{p_3}{p_2} \qquad \cos \mathrm{B} = +\frac{p_1}{p_2}$$

In these formulæ a, b, and q_2 are to be replaced by the values obtained from the spherical triangle (fig. 3) as functions of θ, p_1, p_2, p_3. That this is a contact transformation is evident from the following equations, which are the result of differentiation of the

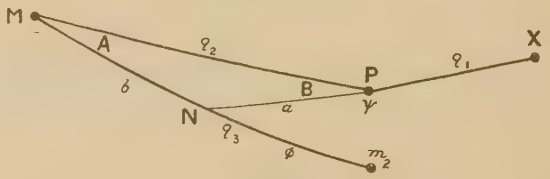

FIG. 3.

above equations. The quantities differentiated are supposed to be expressed as functions of ψ, θ, ϕ and p_1, p_2, p_3.

$$\frac{\partial q_3}{\partial \theta} = \frac{\partial \Theta}{\partial p_3} \qquad \frac{\partial q_3}{\partial p_2} = \frac{\partial q_2}{\partial p_3} \qquad \frac{\partial q_3}{\partial p_1} = \frac{\partial q_1}{\partial p_3}$$
$$\frac{\partial q_2}{\partial \theta} = \frac{\partial \Theta}{\partial p_2} \qquad \frac{\partial q_2}{\partial p_1} = \frac{\partial q_1}{\partial p_2} \qquad \frac{\partial q_1}{\partial \theta} = \frac{\partial \Theta}{\partial p_1}$$
$$\frac{\partial q_3}{\partial \phi} = \frac{\partial \Phi}{\partial p_3} \qquad \frac{\partial q_1}{\partial \psi} = \frac{\partial \Psi}{\partial p_1}$$

The other differential quotients are either zero or not required.

The transformation reduces the complicated expressions for F_1 and F_2 to the simple formulæ

$$(2)\quad \mathrm{F}_1 = (a_3^2 - p_3^2)^{\frac{1}{2}} \sin (q_3 + l_2) \text{ and } \mathrm{F}_2 = (a_3^2 - p_3^2)^{\frac{1}{2}} \sin q_3.$$

These and $\Phi = p_3$ (in first line) have to be substituted in (1). Neither the time nor p_1 nor q_1 nor q_2 occur in H (R, r, q, p), and hence $\mathrm{H} = a_1$, $q_1 = \beta_2$, $p_1 = \alpha_2$ (as in section 2), $p_2 = a_3$, and $\cos \mathrm{B} = a_2/a_3$ where a, β designate canonic constants. The value of p_2 ($= a_3$) is already entered in formula (2). MP (fig. 3) is therefore the invariable plane. H contains thus only q_3 and p_3 in addition to the $6n - 12$ variables r_k, l_k, b_k, R_k, L_k, B_k, and the problem is of the order $6n - 10$.

In the *3-body*-problem the sums $\sum_{3}^{n-1}$ in (1) become zero and the problem is of the eighth order; in the co-planar 3-body-problem the second and fourth lines in (1) disappear, as also all the q and p variables, since $\Phi = p_3$ now obtains a constant value. The co-planar 3-body-problem is thus of the sixth order.

Crieff:
1920 *July* 7.

Note on Equal-Area Maps. By F. Bowman, M.A.

(*Communicated by the Secretaries.*)

It is known that all those equal-area representations of the plane (X, Y) on the plane (x, y) in which the parallel straight lines $Y = \text{const.}$ correspond to the parallel straight lines $y = \text{const.}$ are given by

$$x = \frac{X}{\phi'(Y)} + \omega(Y),$$
$$y = \phi(Y),$$

where ϕ and ω are two arbitrary functions.*

The simplest equal-area representation of a sphere, known in principle to Archimedes, is that which is obtained by projecting the points of the sphere on to a tangent cylinder, the rays of projection passing through the axis of the cylinder at right angles, and then developing the cylinder into a plane. In the case of the earth (latitude u, longitude v), the equator being the great circle of contact between the earth and the cylinder, the equations of this projection are

$$X = v,$$
$$Y = \sin u.$$

It follows that all those equal-area maps of the earth in which the parallels of latitude $u = \text{const.}$ become the parallel straight lines $y = \text{const.}$ are included in the formulæ

$$x = \frac{v}{\phi'(\sin u)} + \omega(\sin u),$$
$$y = \phi(\sin u);$$

and these two equations may be replaced by

$$x = vf'(y) + w(y) \quad . \quad . \quad . \quad . \quad (1)$$

where f and w are arbitrary functions, together with the relation

$$f(y) = \sin u,$$

which enables us to graduate the y-axis in terms of u.

* *Vide* Scheffers, *Theorie der Curven*, p. 123; or Gravé, Liouville, 1896, p. 320.

We may interpret (1) by saying that any two meridians, say $v=0$, $v=V$, may be represented by any two curves on the map. Or we may suppose that any two curves on the earth may have arbitrary representations on the map. For example, any two curves on the earth, say two coast-lines (which in the simplest case will be cut only once by any parallel of latitude), may be made to correspond to the two straight lines $x=0$, $x=a$, and the portion of the surface of the earth enclosed by them may in this way be mapped on a rectangle, the relative proportions of all areas being conserved.

Thus if

$$v=\chi(\sin u)$$

be one such curve, it will be represented on the map by

$$x=\chi[f(y)]f'(y)+w(y);$$

and if we suppose that f is known, and we take $w(y)$ to be

$$-\chi[f(y)]f'(y),$$

this curve will become the straight line $x=0$, and (1) will be

$$x=[v-\chi(f)]f'(y).$$

Let any other such curve be $v=\psi(\sin u)=\psi(f)$. Its representation will be

$$x=[\psi(f)-\chi(f)]f'(y);$$

that is,

$$x=a,$$

provided $f(y)$ is found from

$$\int[\psi(f)-\chi(f)]df=ay+b.$$

It may further be remarked that an equal-area map can be constructed in which a given coast-line will have (to an eye placed in a suitable position) its correct shape as viewed from the centre of the earth. We may suppose the coast-line to be divided into two parts (say by the two points where it touches its limiting parallels of latitude). Then the two functions f and w must be chosen so that each part will have the shape which it would have on a gnomonic chart.

The Long-Period Terms in the Growth of Trees.
By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *M.N.*, 80, 619, reference is made to *The Climatic Factor*, by Ellsworth Huntington, as containing valuable data for thousands of years. I have not yet been able to see a copy of this work, in spite of inquiries in America: but Mr. Jeffreys kindly lent me a copy of *Climatic Cycles and Tree-growth*, by A. E. Douglass (Carnegie Institution of Washington, 1919), which contains similar material—material which must be of unique value for the study of the past. There are records back to B.C. 1306, though this earliest group is compiled from only 1 to 4 trees (down to B.C. 260). From B.C. 274 to 1914, however, we have a complete record from 11 trees. I lost no time in analysing this splendid record.

2. To avoid possible difficulties connected with the discontinuity near B.C. 270, attention was concentrated in the first instance on the homogeneous series for 11 trees from B.C. 274 to A.D. 1914. Mr. Bellamy kindly took 10-year sums for the study of the long-period terms. There is a notable drop in these sums as we pass from ancient to modern times, which it was decided to remove (approximately) before applying harmonic analysis. To do this the logarithms were taken, and were corrected as follows:—

Date.	10-year Sum.	Log.	Corr^{n}.	Resid.
− 115	23·43	1·369	− ·900	469
+1885	11·19	1·049	− ·600	449

In 2000 years the correction is assumed to change uniformly by ·300 or log 2, so that the rate of growth (as tabulated by Mr. Douglass) was reduced to one-half, and the corrections restore a uniform though fictitious scale as more convenient for considering fluctuations.

3. The 216 residuals in the last column range over about 400 of the units there adopted (viz. ·001 in the logarithm). [There are really 218 of them, but as 216 is a very convenient number to divide, the first two were omitted and the series restricted to − 245 to + 1905.] The harmonic terms were calculated as in Table I. for the Fourier divisors 2 to 16, which cover the range of special interest to us. The zero epoch is − 245.

4. In forming this table, the corrected logarithms for 3 consecutive sums of 10 years were added together to shorten the arithmetic. The results are thus only approximate, but should be of the right order of magnitude to indicate possible periodicities. The figures for the early divisors 2, 3, and 4 are only inserted for completeness, and cannot be taken as indicating real periodicities without further evidence. After these the divisors 8 and 11, periods 270 and 196 years, offer the best indications, though there

is nothing large enough to be convincing. If there are really two periodicities of these approximate lengths, their coexistence may explain some of the difficulties already encountered in identifying a single long period in various phenomena. The one near 270 years is sufficiently close to the 260 years already found (*M.N.*, **79**, 531, and **80**, 619) to suggest further investigation, though on the evidence of Table I. as it stands the period should apparently be longer still, say 280 years at least, because of the change of sign between divisors 7 and 8 (*M.N.*, **73**, 716). We have already had

TABLE I.

Fourier Sequence of Corrected Logs for 10 *Years: Sequoias B.C.* 270–*A.D.* 1810.

Divisor.	Period.	R sin θ_0.	R cos θ_0.	R^2.
2	1080	−·006	+·054	295
3	720	+·034	+·034	232
4	540	+·034	−·006	119
5	432	−·018	−·013	49
6	360	−·005	−·008	9
7	309	+·019	+·011	48
8	270	−·005	−·026	70
9	240	−·016	−·002	33
10	216	+·016	·000	32
11	196	+·005	−·026	70
12	180	−·018	·000	32
13	166	−·010	−·009	18
14	154	+·008	·000	6
15	144	+·010	+·010	20
16	135	−·005	−·002	3

experience of the fact that such changes of sign may be only accidental, and that further scrutiny is necessary to get the real period; nevertheless let us adopt, as an independent check, a period of 290 years for examination.

5. In Table II. are given the calculated coefficients of sin θ and cos θ for single periods of this length, for initial dates shown in the first column. Mr. Douglass's independent series of 1 to 4 trees before B.C. 270 gives three more complete cycles which have been added above the horizontal line. From the deduced coefficients of sin θ and cos θ (viz. R sin θ_0 and R cos θ_0 shown in the second and third columns) the values of θ_0 in the expression R cos $(\theta - \theta_0)$ are deduced in the fourth column. They show no great consistency, nor can we reasonably expect it when we already

know that there is another period of about 200 years comparable with that under investigation. To eliminate its disturbing effects we must take 2 of our 290-year periods together, equal approximately to 3 of the other; and when this is done in the fifth and

TABLE II.

Examination of Possible Period of 290 Years.

Initial Date.	R sin θ_0.	R cos θ_0.	θ_0.	Means of 2: R sin θ_0.	Means of 2: R cos θ_0.	θ_0.	$\theta_0 - \theta_M$.	θ_1.	R.
			°			°	°	°	°
−1135	−·050	+·103	334						
− 845	−·023	−·073	197	−·036	+·015	293	+107	+75	40
− 555	+·007	+·015	25	−·008	−·029	196	+ 10	−14	30
− 265	+·020	−·068	164	+·014	−·026	151	− 35	−51	30
+ 25	+·025	+·032	38	+·022	−·018	129	− 57	−65	28
+ 315	+·013	−·128	174	+·019	−·048	158	− 28	−28	52
+ 605	+·023	+·018	51	+·018	−·055	162	− 24	−16	58
+ 895	+·032	−·030	133	+·028	−·006	102	− 84	−68	29
+1185	−·028	−·061	206	+·002	−·046	177	− 9	+15	46
+1475	+·002	+·020	6	−·013	−·020	213	+ 27	+59	24
Mean	+·003	−·017		+·003	−·027	186			

sixth columns, the value of θ_0 in the seventh column shows fair persistence. The first value is discordant, as shown by the eighth column, where the mean value $\theta_M = 186°$ is subtracted. We can reduce the discordance below 90° by altering the period to 284 years, which gives a correction of 8° per cycle, as in column θ_1. But there is still something systematically wrong with the phases.

6. Let us at this point turn to the other suggested periodicity of about 200 years. On arranging the numbers for the years −270 to 1830 in 10 sets of 20 years each, and adding up corresponding sets, the following means were obtained, the unit being 0·01 in the logarithm—

30 28 26 28 34 32 28 28 30 29.

Now it is at once obvious that the half-cycle of about 100 years is at least as important as that of 200 years. Writing the second half under the first, and taking means, we get

1st half . .	30	28	26	28	34
2nd half . .	32	28	28	30	29
Mean . . .	31	28	27	29	32

7. This century term was found by Mr. Douglass, using his mechanical process, and seems worth immediate consideration before we return to the longer periods. To study it the first

harmonic was first calculated for each century separately as in Table III. But as the accidental errors are considerable, means were taken for each three consecutive years and means again of each three of the new series, thus forming the "smoothed" columns, from which R and θ are calculated in the next columns.

TABLE III.

Harmonic Coefficients for the Century Term in Units of 0·001 *in the Logarithm.*

Initial Date.	R sin θ.	R cos θ.	Smoothed.	R.	θ.	θ_c.	$\theta - \theta_c$.
					°	°	°
−1265	−364	−222					
−1165	+ 80	+ 40					
−1065	+ 24	− 24	−31 −41	51	217	266	−49
− 965	− 86	− 78	−14 −29	32	205	267	−62
− 865	+ 28	+ 4	−14 −27	30	208	268	−60
− 765	− 50	− 30	−13 −19	23	215	269	−54
− 665	+ 40	− 12	0 − 6	6	180	270	−90
− 565	− 20	+ 10	− 8 + 2	8	286	271	+15
− 465	− 4	+ 22	−19 +21	28	318	272	+46
− 365	− 38	+ 2	−33 +30	44	312	273	+39
− 265	− 86	+114	−33 +41	52	321	274	+47
− 165	+ 14	− 22	−11 +34	36	343	275	+68
− 65	+ 8	+ 50	− 3 +34	35	355	276	+79
+ 35	+ 56	+ 46	0 +22	22	360	277	+83
+ 135	−102	− 8	−34 + 2	34	273	278	− 5
+ 235	+ 6	− 6	−47 −20	51	248	279	−31
+ 335	−136	− 90	−66 −31	73	245	280	−35
+ 435	− 24	− 8	−44 −19	48	246	281	−35
+ 535	− 50	+ 26	−32 − 6	32	259	282	−23
+ 635	+ 42	− 16	− 9 − 1	9	264	283	−19
+ 735	− 40	+ 6	−16 − 2	17	262	284	−22
+ 835	− 2	− 16	−21 − 6	22	253	285	−32
+ 935	− 58	0	−33 − 4	33	263	286	−23
+1035	− 26	− 4	−34 − 2	35	267	287	−20
+1135	− 24	− 2	−32 + 4	32	276	288	−12
+1235	− 60	+ 16	−30 + 7	31	283	289	− 6
+1335	+ 12	+ 14	−22 +14	26	302	290	+12
+1435	− 44	− 6	−21 +17	27	308	291	+17
+1535	− 8	+ 66	− 9 +19	21	334	292	+42
+1635	− 6	− 8					
+1735	+ 32	− 16					

8. The reasons for adopting θ_e will appear in a moment. Accepting it as representing the appropriate correction to the period under investigation, this becomes

$$100 \times 360/359 \text{ years} = 100{\cdot}3 \text{ years}.$$

Douglass found 101 years, and we shall presently find some confirmation of this rather larger value: but at this point the increase of 1° shown in θ_e seemed to satisfy requirements.

9. The residual differences $\theta - \theta_e$ show fluctuations which suggest the interference of another periodicity. The general effect of such interference is well known. If we put

$$a \cos \phi + b \cos (\phi - \epsilon) = \mathrm{R} \cos (\phi - \mathrm{E}),$$

then

$$\mathrm{R}^2 = a^2 + b^2 + 2ab \cos \epsilon,$$

and

$$\tan \mathrm{E} = \frac{b \sin \epsilon}{a + b \cos \epsilon}.$$

The coefficient R thus has minima when $\epsilon = (2n+1)\pi$, and maxima when $\epsilon = 2n\pi$, at all of which E vanishes. The column $\theta - \theta_e$ suggests that these critical points occur near the dates

Date.	−1400.	−600.	+200.	+1000.	+1800.
ϵ	0	π	2π	3π	4π
R	Max.	Min.	Max.	Min.	Max.

and though these dates are only rough, the general nature of the fluctuation cannot be far wrong. The choice of θ_e can now be explained: it was taken so as to make the maxima and minima of R correspond to zero values of $\theta - \theta_e$ as nearly as possible.

10. Thus we have another periodicity interfering in about $16 \times 100{\cdot}3$ years, and its length is thus $1605/17 = 94{\cdot}4$ years. Now $3 \times 94{\cdot}4 = 283{\cdot}2$, and we can thus with some plausibility identify the interfering wave as the third harmonic of the term already found in § 5, to which the period 284 years was tentatively assigned. But this periodicity was apparently itself subject to interference, and it is natural to inquire whether perhaps the disturbance comes from a term of 300·9-years period, the triple of 100·3 years. Terms of 300·9 and 283·2 years will interfere in

$$4814 \text{ years} = 16 \times 300{\cdot}9 = 17 \times 283{\cdot}2,$$

and we should thus expect changes of sign in the E of § 9 every $4814/2 = 2457$ years, which is nearly the whole length of the series tabulated in Table II. This fits the facts we there found, for though the terminal values of θ_1 are not zero, we can make them so by subtracting (say) 45° from θ_1 throughout, now that we have some warrant for doing so. In the middle of the series the values of θ_1 will then approach −90°, indicating that the coefficients of the two interfering terms are nearly equal in value.

11. By a few trials it was not difficult, from the general considerations now available, to represent the two interfering terms

for Table II. to a very fair approximation. But before giving the results one more remark may be made. If we look down the column for R in Table III., we have a well-observed maximum near the middle (M), while at the beginning and end of the series R is approaching two other maxima B and D, not quite reached in either case, the assigned dates being −1400 and +1800; but there are sufficient terms to show pretty clearly that M is bigger than D and less than B. Thus there is a steady diminution in the extent of the fluctuations. It must be remembered that we are dealing with logarithms, and not with the growths themselves, so that the diminution is real and not illusory; the actual *percentage* effect is less.

12. Is this due to a diminution in the efficient cause? or is it merely that, while the cause itself remains constant, the response of the tree grows weaker with age? From several directions it was suggested that the latter is the right alternative, and it is a natural and reasonable one. We shall adopt it for use; and we can thereby explain, for instance, why the values obtained for R shown in the last column of Table II. seem to remain nearly constant. If there are two interfering terms, R should vary: it should be smaller at the beginning of the series, when the component periodicities are opposed, than at the end, when they are in sympathy. When we find that R nevertheless remains nearly constant, the explanation probably is that at the later date the components are both small and the *sum* of the two is but little larger than the *difference* at the beginning of the table when they are opposed but large.

TABLE IV.

Analysis of the Results of Table II. into two Components.

(The unit is 0·001 in the logarithm.)

a.	ϕ.	$-a\sin\phi$.	$-a\cos\phi$.	Table II.	Sums.	b.	$\phi-\epsilon$
62	234°	+50	+37	−36 +15	+14 +52	54	15°
56	225	+40	+40	− 8 −29	+32 +11	34	71
50	216	+30	+40	+14 −26	+44 +14	46	72
44	207	+20	+39	+22 −18	+42 +21	47	64
38	198	+12	+36	+19 −48	+31 −12	33	111
32	189	+ 5	+32	+18 −55	+23 −23	33	135
26	180	0	+26	+28 − 6	+28 +20	34	55
20	171	− 3	+20	+ 2 −46	− 1 −26	26	182
14	162	− 4	+13	−13 −20	−17 − 7	18	247

13. In the fifth and sixth columns of Table IV. we repeat from Table II. the values of what were there denoted by $R\sin\theta_0$ and $R\cos\theta_0$, but what are now regarded as

$$a\sin\phi + b\sin(\phi-\epsilon) \text{ and } a\cos\phi + b\cos(\phi-\epsilon)$$

In columns 2 and 3 are given the reversed values $-a \sin \phi$ and $-a \cos \phi$, calculated with values of a and ϕ given in columns 1 and 2. Thus the sums in the seventh and eighth columns eliminate the ϕ term and leave "observed" values of $b \sin (\phi - \epsilon)$ and $b \cos (\phi - \epsilon)$, from which b and $(\phi - \epsilon)$ are derived in columns 9 and 10. Looking first at $(\phi - \epsilon)$ we see that there is a steady increase, with accidental errors which are fairly small until we approach the end of the series, when the coefficient b is small. To diminish accidental errors let us take means of three consecutive lines; we then get

TABLE V.

$b \sin (\phi-\epsilon)$.	$b \cos (\phi-\epsilon)$.	b.	$(\phi-\epsilon)$.	a.	$a-b$.
+30	+26	+40	49°	56	+16
+32	+ 5	+32	99	38	+ 6
+ 3	− 4	+ 5	142	20	+15

14. The phases $(\phi - \epsilon)$ show an increase of 93° in 6 cycles or 16° per cycle, corresponding to a period of $290 \times 360/344 = 304{\cdot}5$ years.

Different methods of estimating the increase would assign periods differing a little from this, but not much. It is sufficiently near the triple of the period found in § 8 to make identification reasonable; and at the same time it is suggested that a slightly longer period, say 101 years, as found by Mr. Douglass, with 303 years for the triple, would perhaps fit all the facts better.

15. The coefficients b show a diminution as before. If we compare them with the adopted values of a we see that b is on the average 9 units (*i.e.* ·009 in the logarithm) less than a. A constant *difference* $a - b$, seems a more appropriate conception than a constant *ratio* of a to b since we are dealing with logarithms already. The fact that the difference $a - b$ is found smaller in the later terms than in the first may be interpreted to mean that too rapid a diminution of a has been assumed in Table IV. At the top of the table ϕ and $\phi - \epsilon$ are in opposite phases, so that observation gives the difference $a - b$ directly. We can at these dates increase a and b together, or diminish them together, but we cannot alter the difference. At the bottom of the table, on the other hand, observation gives their sum $a + b$, and if we increase one we must diminish the other. The fact that they are both small, however, increases accidental errors and the effect of the (presumably anomalous) -6 for $R \cos \theta_0$ at date +895 is unduly felt unless we dilute it by taking several terms together. Let us take the mean of the last five terms; then assigning different values for a without altering ϕ we get

TABLE VI.

a.	$-a \sin \phi$.	$-a \cos \phi$.	Table II.	Sums.	b.	$\phi-\epsilon$.	$a-b$.
26	+2·0	+25·4	+10·8 − 35·0	+12·8 − 9·6	16·0	127	+10
31	+1·8	+30·4	+10·8 − 35·0	+12·6 − 4·6	13·6	110	+17
36	+1·2	+35·2	+10·8 − 35·0	+12·0 + 0·2	12·0	90	+24

16. The top line represents the solution already given in Table IV., where a decreases 6 units each cycle. The next lines show the consequences of assuming the decrease to be 5 units and 4 units respectively. The last line clearly gives $a-b$ too large, and the phase $\phi-\epsilon$ too small, the suggested value of the latter being 118° for a period of 301 years or thereabouts. The mean between the first two lines satisfies the conditions for both $a-b$ and $\phi-\epsilon$, and will be adopted. It seemed worth while to give this argument at some length, since the material, though it may at first have seemed too slight to justify careful study, turns out to be unexpectedly sensitive to departures from the correct suppositions, if we may regard those now obtained as approximately correct. The revision of Table IV. now stands as follows:—

TABLE VII.

Revised Analysis of Table II. into two Components.

(Unit 0·001 in the logarithm.)

a.	ϕ.	$-a\sin\phi$.	$-a\cos\phi$.	Table II.	Sums.	b.	$\phi-\epsilon$.	C.	O−C.
	°						°	°	°
62	234	+50	+37	−36 +15	+14 +52	54	15	30	− 15
56	225	+40	+40	− 8 −29	+32 +11	34	71	46	+ 25
51	216	+30	+41	+14 −26	+44 +15	46	71	62	+ 9
45	207	+21	+40	+22 −18	+43 +22	47	63	78	− 15
40	198	+12	+38	+19 −48	+31 −10	33	108	94	+ 14
34	189	+ 5	+34	+18 −55	+23 −21	30	132	110	+ 22
29	180	0	+29	+28 − 6	+28 +23	36	(51)	126	− 75
23	171	− 4	+23	+ 2 −46	− 2 −23	23	185	142	+ 43
18	162	− 6	+18	−13 −20	−19 − 2	19	264	158	+106

17. Under heading C is given a phase increasing at the rate of 16° per cycle of 290 years, which seems to satisfy the observations until the coefficient b becomes small at the end of the series. The corresponding period is 303·5 years. One-third of this is close to Mr. Douglass's value, 101 years.

18. We now return to the 200-year term shown in Table I. and mentioned in § 6, but put aside for the moment in order to investigate the more striking 100-year term, which is beyond the limits of Table I. The mean values of § 6 give a coefficient −·012 for the 200-year term. But as two interfering terms have been suggested in other cases, it seems better to make a detailed analysis. This was done for every set of 200 years, taking 10 years as the unit, and the results are shown in the second and third columns of Table VIII. As they stand they are liable to disturbance by the 300-year term: and means of three consecutive terms are therefore taken in the fourth and fifth columns. This single process seemed sufficient without repeating it, as in Table III. The phases θ were then compared with θ_c which decreases uniformly 21° per cycle, and corresponds to a period of $2\times 94{\cdot}4$ years (see § 10). The differences $\theta-\theta_c$ show that there is again interference, which is confirmed by the two minima in R; and the cycle of interference is about 1400 years. Now this is something new.

TABLE VIII.

Harmonic Coefficients for the 200-year Term, in Units of ·001.

Initial Date.	R sin θ.	R cos θ.	Means of 3.	θ.	θc.	θ − θc.	R.
−1265	− 49	−291					
−1065	−104	+ 62	−73 −63	229°	316°	−87°	97
− 865	− 67	+ 40	−59 +49	310	295	+15	76
− 665	− 5	+ 45	−29 +29	314	274	+40	41
− 465	− 16	+ 1	−24 + 6	283	253	+30	25
− 265	− 50	− 29	− 8 −14	208	232	−24	16
− 65	+ 43	− 13	+11 −25	156	211	−55	27
+ 135	+ 41	− 34	+21 −58	160	190	−30	62
+ 335	− 20	−127	−10 −69	188	169	+19	70
+ 535	− 50	− 47	−26 −59	204	148	+56	65
+ 735	− 8	− 4	− 8 −31	195	127	+68	32
+ 935	+ 33	− 41	+ 2 − 5	158	106	+52	6
+1135	− 19	+ 30	+18 +13	53	85	−32	22
+1335	+ 39	+ 51	+ 5 +25	11	64	−53	26
+1535	− 6	− 7					

If the interfering component were one of 202 years, as we might expect from analogy with the 101- and 303-year components, then the cycle of interference should be 2900 years, just double that indicated in the table. Hence the other component must be near 220 years.

19. It is not difficult to give an analysis showing the two terms, as in Table IX. The typical notation of § 9 is adopted here also. The coefficient a and argument ϕ in the first two columns refer to the 189-year term. The gradual diminution of a was assumed empirically, but seems to be about right. The argument ϕ is that denoted by θ_c in Table VIII. and corresponds to a period of 189·8 years.

TABLE IX.

Separation of 189-year and 220-year Terms.

a.	ϕ.	$-a\sin\phi$.	$-a\cos\phi$.	Table VIII.	Sums.	$\phi-\epsilon$.	C.	O−C.
60	316°	+41	−43	−73 −63	−32 −106	198°	317°	−119°
56	295	+51	−24	−59 +49	− 8 + 25	342	350	− 8
52	274	+52	− 4	−29 +29	+23 + 25	43	23	+ 20
48	253	+46	+14	−24 + 6	+22 + 20	47	56	− 9
44	232	+35	+27	− 8 −14	+27 + 13	64	89	− 25
40	211	+21	+34	+11 −25	+32 + 9	74	122	− 48
36	190	+ 6	+35	+21 −58	+27 − 23	130	155	− 25
32	169	− 6	+31	−10 −69	−16 − 38	203	188	+ 15
28	148	−15	+24	−26 −59	−41 − 35	230	221	+ 9
24	127	−19	+14	− 8 −31	−27 − 17	237	254	− 17
20	106	−19	+ 6	+ 2 − 5	−17 + 1	279	287	− 9
16	85	−16	− 1	+18 +13	+ 2 + 12	369	320	+ 49
12	64	−10	− 5	+ 5 +25	− 5 + 20	346	353	− 7

20. The result of subtracting $a \sin \phi$ is shown in the seventh and eighth columns, and the deduced values of $\phi - \epsilon$ correspond, as shown by the column C, to a period of 220 years, which advances 33° per cycle on the adopted 200 years. The question now arises whether after clearing out both the 189- and 220-year terms, a small periodicity of 202 years might not emerge from the residuals O − C; but the answer seems to be in the negative.

21. The first term in O − C, viz. −119°, is discordant. By referring to Table VIII., columns 2 and 3, we see that it arises from a specially large and discordant first group. We have found in all the cases studied that the hypothesis of a gradually diminishing response from the tree to the efficient cause, whatever it may be, fits the facts. A corollary is that in early life the tree may be specially sensitive, and one particular "accidental" climatic feature might leave such an impress on its growth (shown in the form of a bulge on the growth curve) as to displace the maximum of all other fluctuations. Moreover, the earliest records are based on measures of one tree only, though three others are associated later: these seem to be good reasons for not attaching too much importance to the anomalous first term.

22. The coefficient of the 220-year term has not yet been examined. We have denoted this term in Table IX. by $b \cos(\phi - \epsilon)$, and b^2 can, of course, be found by summing the squares of the seventh and eighth columns. If we multiply the deduced value of b by cos (O − C) we get the series of values—

(−54) 26 33 29 27 21 33 39 63 30 17 8 21

Omitting the discordant first term, there is here no noticeable diminution until close to the end: indeed, if we omit the last three terms, there is an actual rise, which cannot be avoided by any reasonable change in the first (and most doubtful) column of Table VIII. It may be that this 220-year term is in a class by itself; but the diminution does actually appear in the last few terms, and the delay in its appearance may be accidental only.

23. Collecting the results so far found for long-period terms we have—

TABLE X.

Collected Results for Long-Period Terms in Trees.

Period.	Coefficient.	Maxima.	Min.
303 years	·055 − ·0020(t + 12·65)	−1123; +1907	1755
284 ,,	·069 − ·0020(t + 12·65)	−1225; +1899	1757
220 ,,	·029	−1311; +1769	1879
189 ,,	·064 − ·0020(t + 12·65)	−1278; +1746	1840
101 ,,	·050 − ·0020(t + 12·65)	−1315; +1715	1765
94·4 ,,	·040 − ·0020(t + 12·65)	−1300; +1721	1768

The values given under the heading "coefficient," with their rates of decay, must be regarded as rough estimates merely, especially those for the century terms. The time t is here measured in centuries from A.D. 0. The century terms are inserted for reference, but attention is at present directed chiefly to any terms which may by any possibility appear as of about 250 years in length.

24. There is an independent check which we may apply at this point. On p. 113 of his book Mr. Douglass gives a series of "Flagstaff 500-year measures: 2 to 19 trees," which provides 51 complete sums for 10 years, the first being denoted by 1405 A.D. and the last by 1905 A.D. It is not, of course, possible to identify separately the 303- and 284-year periodicities from a series so short as 500 years in length; but Table X. shows that they have a common minimum at about 1756, and this should be discernible in the Flagstaff series if the periodicity is real. To test this the logarithms of the 51 ten-yearly sums were first corrected for an obvious secular term by the formula

$$-0{\cdot}8000 - {\cdot}00409n + {\cdot}000274n^2,$$

where n is the number of the term from 1405 taken as zero. Thus for date 1505 we have $n = 10$, and the logarithm 0·9934 (for sum 9·85 mm.) is corrected thus:

$$0{\cdot}9934 - 0{\cdot}8000 - {\cdot}0409 + {\cdot}0274 = 0{\cdot}1799.$$

25. Further, there is a very noticeable period of 150 years, to which attention has been called by Mr. Douglass and others following him. Reference to Table I. of this paper will show that there is no large term of this length in the sequoias; but in the Flagstaff series it is so large as to swamp all others. By harmonic analysis we obtain the following terms, the coefficients being in units of 0·001 in the logarithm:—

$$+42 \sin\theta + 55 \cos\theta - 5 \sin 2\theta + 40 \cos 2\theta + 19 \sin 3\theta + 28 \cos 3\theta,$$

maxima of the first harmonic falling in 1420 and 1570.

26. After correcting for these terms the residuals were examined for a term of 288 years, which falls between 284 and 303 years, and has the convenient advance of 12°·5 every 10 years. The deduced term is very small, viz.:—

$$-8 \sin\theta - 2 \cos\theta = +10 \cos(\theta - 256°),$$

but the maximum does fall in 1898 and minimum in 1754, close to the dates of Table X.

27. It was natural now to inquire whether the Flagstaff 150-year term also appears in the sequoias, though according to Table I. the coefficient must be small. Each set of 150 years was analysed separately, but nothing much is gained by giving these rather erratic numbers. In Table XI. they are combined into sets of 6, each set having 4 cycles in common with an adjacent set.

TABLE XI.

Possible 150-Year Term in Sequoias.

Initial Date.	$R \sin \theta_0$.	$R \cos \theta_0$.	R.	θ_0.	Max.	C.	O−C.
− 715	+ 2	+11	12	10°	− 411	− 398	−13
− 415	+ 5	+ 6	8	40	− 98	− 95	− 3
− 115	+18	+11	21	59	+ 210	208	+ 2
+ 185	+ 9	+ 4	10	66	+ 513	511	+ 2
+ 485	+18	− 1	18	93	+ 824	814	+10
+ 785	− 2	−19	19	186	+1163	1117	+46
+1085	− 4	− 6	7	214	+1474	1420	+54
+1385	−22	−11	25	244	+1787	+1723	+64

28. The coefficients of $\sin\theta$ and $\cos\theta$ in the expression $R\cos(\theta - \theta_0)$ are given in the second and third columns, and the deduced values of R and θ_0 in the fourth and fifth. From θ_0 we can deduce a date of maximum for about the middle of the group of 6 cycles, as in the sixth column. The mean of the first five maxima gives the date 208, not far from the 210 deduced from the middle series. Now the Flagstaff 500-year series assigns a well-determined maximum to date 1570 (as in § 25). If we determine the period from these two maxima we have—

$$1570 - 208 = 1362 = 8 \times 170 = 9 \times 151{\cdot}3 = 10 \times 136.$$

There can be no hesitation in selecting from these 151·3 years as the period, which is close to the half of 303. Let us assume, then, that of the two possible second harmonics which our previous results suggest, viz. 142 and 151·5 years, the latter predominates in the sequoias though its coefficient is small: while in the Flagstaff 500-year series it has for some reason a large coefficient. Adding (and subtracting) multiples of 303 years to 208, we get the dates C in the seventh column of Table XI.

29. Now the differences O − C show a steady increase which cannot be blinked. If we determine the period from them alone, we get 157·5 years, and this is distinctly against the above interpretation. In the first instance it was accepted as conclusive; but on reconsideration it was determined to state the alternative, which fits in with other evidence. A period of 157·5 years will not fit in with anything else; and it assigns a maximum to the sequoias when there is a minimum of the Flagstaff 500 years. The only thing to be said for it is that it accords with the differences O − C in Table XI.; and after all they must not be treated too seriously. Though consistent, they are small, and they relate to terms with very small coefficients. Their consistency may quite reasonably be a mere accident.

30. We have at present scarcely any other means of checking the 150-year term or terms. There are two other series of some length among Mr. Douglass's records:—Pp. 116, 117: Windsor, Vermont: 11 trees: 1651–1912. This series has unfortunately a sharp discontinuity about 1800: the mean annual advance before 1800 is about 0·60, and afterwards 1·75. Any inferences about long-period terms depend essentially on the treatment of this discontinuity. There are, however, nearly 150 years before the discontinuity occurs; and the assigned maximum is about 1745; the remainder of the series suggests that this is perhaps rather late: 1730 would not be far wrong, and is not far from the 1723 of Table XI. P. 117: Oregon: 17 trees: 1710–1912. This series shows a rapid fall, the logarithm diminishing by about ·00176 every year. Making this allowance, there is a 150-year term with coefficient about ·040 in the logarithm and maximum at about 1830. This should be 1874 and is not in good accord.

31. Some of the discordances noticed may, however, be due to the interference of the 142-year term (the second harmonic of the 284-year term). It might be supposed that the association of a term of shorter period would necessarily cause an apparent shortening, and could not cause an apparent lengthening of the period. When we have two interfering terms the mean resultant period is that of the term with the larger coefficient, which we have identified as that of 151·5 years in this case. With the notation of § 9, we assume that a is the larger coefficient, so that $a+b\cos\epsilon$ never vanishes. The mean period is then that of ϕ; but as E oscillates, the change of $\phi-\mathrm{E}$ is sometimes greater and sometimes less than that of ϕ. We have

$$\frac{d\mathrm{E}}{d\epsilon}=\frac{b(a\cos\epsilon+b)}{a^2\sin^2\epsilon+(a\cos\epsilon+b)^2}.$$

We are at present considering the effect of an interfering term of period shorter than 151 years, so that ϵ is negative and $(\phi-\epsilon)$ increases more rapidly than ϕ. We see that $(\phi-\mathrm{E})$ will also increase more rapidly than ϕ so long as $(a\cos\epsilon+b)$ is positive—that is, for more than half the cycle of interference; but in the remainder more slowly, so that the period will appear to be longer than 151 years. When b is nearly equal to a this remainder is small, and in the limit when $b=a$ it produces the well-known jump of half a wave-length at the dark fringes. But when b is small the values of ϵ satisfying the condition $a\cos\epsilon+b=0$ are not far from $\pm 90°$, and the partition of the cycle is nearly equal. A small term of 142-years period will cause the period of 151 years to be diminished for more than half the cycle of interference (about 2400 years)—that is, for more than 1200 years: but for the remainder the period will appear longer than 151 years. The ratio of b to a settles both the partition of the interference cycle and the maximum disturbance, and thus we have a test in the reconciliation of the two. There are features in Table XI. which

could be explained in this way: thus between dates 824 and 1163 the period indicated is 170 years, and we might actually obtain this period from the interference of 142 years and 151·5 years by making the coefficient of the former term two-thirds of that of the latter, which would accord with corresponding cases already studied. The resultant period would only remain at this high value for a short time and would only exceed 151·5 years for 640 years in all: for the remaining 1760 years of the interference cycle it would be less than 151·5 years, its lowest value being 148 years. But the fundamental difficulty in Table XI. is that the period never falls below 151·5 years, so that we must either lengthen this value, which is difficult to reconcile with Table II., or do violence to the residuals in some way. It may be of course that the interference of another term (*e.g.* that of 200 years) is in some way responsible. At present it seems better not to attempt a definite solution.

32. The period of 75 years was examined, and there are indications of components near this length, especially of 76 years. The 71-year component is more uncertain.

33. We must now briefly inquire whether these results are reconcilable with what has been formerly obtained in other directions. The longest series previously discussed is that of Chinese earthquake records (1575 years), and the following figures may be recalled from the (more complete) table in *M.N.*, **79**, p. 531.

TABLE XII.

Fourier Sequence of Chinese Earthquakes.

Years.	R^2.	Years.	R^2.	Years.	R^2.	Years.	R^2.
525	85	175	121	105	65	75	26
394	164	158	164	98	8	72	125
315	109	143	225	93	25	69	61
262	485	131	173	88	20	66	25
225	26	121	109	83	2		
197	45	112	5	79	101		

The periodicities of 284 and 303 years fall between two consecutive terms of the above sequence and could not be separated. The figures are quite consistent with their existence. Moreover, when the period of 284 years was tried it was soon seen that there was a conspicuous date recurring in that period. The figures given on p. 532 show the phenomenon quite well, but a special recount shows it better, as in Table XIII., which gives totals for consecutive periods of 20 years. The mean date of each period is obtained by adding the numbers in the top row and first column. Four years are necessarily omitted at the end of each cycle in order to bring corresponding years together, taking 284 years as the period.

TABLE XIII.

Number of Chinese Earthquakes.

Date.	138.	422.	706.	990.	1274.	1558.	Total.
0	12	3	4	2	3	21	45
20	13	2	2	17	10	30	74
40	7	2	2	8	34	26	79
60	2	2	2	12	37	39	94
80	1	2	32	14	31	33	113
100	9	2	5	15	24	13	68
120	2	7	5	1	6	8	29
140	19	3	8	10	6	10	56
160	17	2	8	6	13	6	52
180	11	3	6	12	5	8	45
200	7	2	0	10	27	7	53
220	11	11	6	11	43	...	82
240	7	0	2	3	41	...	53
260	11	7	1	3	14	6	42

There is a conspicuous drop after the 80th year of the cycle, so sharp that several harmonics are required to represent it fully. Moreover, the curve rises to a second peak at the 220th year, so that the second harmonic is even greater than the first. We can thus readily understand the large values of R^2 in Table XII. near 143 years, and again at 72 years. And it is noteworthy that there is a double maximum (72 years and 79 years), which may represent the fourth harmonics of the 284- and 303-year terms.

34. *The Catalogue of Destructive Earthquakes* gave very little support to the suggested 240-year period; but it definitely supports this 284-year period, though the sharp fall seems to occur some 60 years earlier. Possibly the phase varies for different parts of the Earth's surface, being not the same in China as elsewhere. This point must be examined further.

35. Shorter series of observations are incapable of distinguishing between the period of 284 years now suggested and 260 years formerly proposed. For complete separation we require at least 3000 years: and for series of less than 1000 years, one value will do nearly as well as the other. For the Moon less than one cycle is available, so that almost any figure of this order would satisfy the modern lunar observations as such, though other considerations may limit them. At present the evidence of this long series of sequoias would seem to be the best available, and the suggestions it makes must be carefully compared in detail with others. This, however, will require time.

36. Meanwhile if any more trees of great age are available for measurement, or if further measures of value can be made on those already measured, the present evidence suggests an urgent appeal to those in reach of such opportunities to use them.

Summary.

The long series of sequoia measurements (B.C. 1300 to 260 and 274 to 1914) tabulated by Mr. Douglass seem to show clearly a double periodicity, the two components having periods of 303 years and 284 years. The cycle of interference for these is 4840 years $= 16 \times 303 = 17 \times 284$, so that even Mr. Douglass's combined series is not long enough to separate them completely. But there are apparently well-marked third harmonics to both components, interfering in 1613 years $= 16 \times 101 = 17 \times 94{\cdot}5$, which can thus be separated: and with this to guide us we can satisfy the observations of the long-period terms, although there is an additional difficulty arising from the fact that the response of the trees to the efficient cause, whatever it may be, seems to weaken with age.

There is some uncertainty in detecting the second harmonics near 150 years. That of the longer period seems to be the stronger, and is specially strongly marked in the Flagstaff series of 500 years. Combining the well-determined maximum with the mean maximum of early sequoias, a period of 151·5 years is found; and there are (in the sequoia series) indications of interference which may be due to the other term of 142 years; but these terms are weak in the sequoias for some unexplained reason, and the later maxima do not accord with the Flagstaff maxima.

The 284-year period is shown well in Chinese earthquakes, formerly discussed, by a rapid fall in frequency which recurs faithfully. Since 4800 years are required to show the phenomena clearly, it is not surprising that in discussing shorter series a wrong period was assigned. But having now this clue we can see the significance of the double peaks near 100 years and near 75 years in the Fourier sequence for Chinese earthquakes. These double peaks probably represent the separation of the third and fourth harmonics, for which the series is sufficiently long.

The 284-year period is also supported by the *General Catalogue of Destructive Earthquakes*, though the sudden fall seems to occur about 60 years earlier. Possibly the date of this fall varies with locality.

Shorter series already examined are incapable of confirming the values now assigned, but also of contradicting them.

The evidence for two periods of 284 years and 303 years is thus nearly as good as can be at present obtained: but if the opportunities for *long series* of tree measurements are not yet exhausted, an urgent appeal is made to those within reach of them to use them to the full. There seems to be no better chance for exploring the past history of climate and possibly of the earth itself.

Détermination de la Différence de Longitude Greenwich–Paris par transport de l'heure en avion. Note de M. Paul Ditisheim.

(*Communicated by the Astronomer Royal.*)

L'organisation d'un service régulier par avion entre Londres et Paris m'a engagé à tenter un nouvel essai de détermination de la différence de longitude par transport de chronomètres, afin d'établir la limite de précision que permettent d'atteindre les procédés actuels, dans lesquels l'intervalle entre les comparaisons horaires aux stations extrêmes est réduit au minimum possible.

Pour cette expérience, j'ai utilisé un groupe de douze montres de bord, de 63 et 66 m/m de diamètre extérieur, réglées au temps moyen (Deck watches), d'un modèle robuste et maniable, munies de l'échappement à ancre, semblables à celles qui m'avaient déjà servi dans des expériences analogues.* Toutes ces montres venaient d'être simultanément étudiées pendant 43 jours au National Physical Laboratory, à Teddington, suivant le programme d'épreuves de la Classe A. La meilleure d'entre elles avait obtenu le nombre de classement 96·9, non encore réalisé jusque-là ; pour l'ensemble le nombre atteignait 94·9. Les épreuves étant achevées, les montres furent observées encore pendant dix jours à Teddington, dans la position horizontale, à 65·7° Fah^t., et dans une orientation qui a été maintenue autant que possible dans toutes les opérations ultérieures. Transportées à Greenwich, elles furent soumises pendant une semaine, à des séries de comparaisons quotidiennes, en s'aidant du chronographe enregistreur.

Le 18 mai dernier, les douze montres furent placées à bord d'un avion couvert, en partance de l'aérodrome Waddon, Croydon. Elles étaient calées dans leurs boîtes de transport, soigneusement calfeutrées et entourées de couvertures de laine ; le tout fut enfermé dans une valise, dont pendant tout le voyage, je m'efforçai, à l'aide d'un niveau, de maintenir la position horizontale, bien que les observations aux positions, faites à Teddington, eussent révélé des écarts très faibles. Toutes les montres étant munies du balancier Guillaume, leur erreur secondaire est négligeable ; en plus, l'erreur proportionnelle moyenne était inférieure à $0^s{\cdot}01$ par degré.

En raison du mauvais temps et du manque d'essence, ce premier voyage fut d'une durée anormale ; pour quatre voyages ultérieurs, effectués du 21 au 27 mai, la durée moyenne du parcours aérien fut de 2 heures 45 minutes seulement ; le double voyage Paris–Greenwich put même être accompli une fois dans la même journée. Le transport entre les aérodromes de Waddon, Croydon et du Bourget et les observatoires de Greenwich et Paris était fait en automobile.

* Paul Ditisheim, "Essai d'une détermination de différence de longitude par transport de l'heure," *C.R.*, t. cxxxviii., *Les chronomètres de petit format, essai d'une détermination de la différence de longitude Paris-Neuchatel*, Genève, 1905.

Dans tous les voyages, les températures et les pressions furent relevées au départ et à l'arrivée ; un altimètre placé dans la valise enregistrait la courbe des ascensions.

Les observations ont été faites, à Greenwich, par Mr. Bowyer, à Paris par M. Lancelin, qui a bien voulu effectuer l'ensemble des calculs. Les épreuves de Teddington et les comparaisons établissant les "états" ont été exécutées par M. Constable. Je tiens à exprimer ma sincère gratitude aux Directeurs des observatoires susnommés, ainsi qu'à tous ceux dont l'habile collaboration a contribué au succès de l'opération.

Voici les résultats déduits de la moyenne générale des 61 opérations effectuées.

Différence de Longitude Greenwich–Paris, $9^m\,20^s{\cdot}947 \pm 0^s{\cdot}027$.

L'erreur probable est du même ordre de grandeur que celles relevées dans les déterminations faites par les signaux télégraphiques ordinaires.

Comme différence des longitudes mesurées en 1902, MM. Bigourdan et Lancelin avaient publié aux *Annales de l'observatoire de Paris* le chiffre $9^m\,20^s{\cdot}994$, tout en faisant remarquer qu'il serait légitime de supprimer trois séries d'observations extrêmement discordantes des autres et pour lesquelles existait une erreur manifeste.

En supprimant ces trois séries, on a le chiffre .	$9^m\,20^s{\cdot}972$
Du côté anglais, MM. Dyson & Hollis avaient obtenu	$9^m\,20^s{\cdot}932$
La moyenne de ces déterminations donne donc .	$9^m\,20^s{\cdot}952$
Différence du résultat actuel avec la moyenne des résultats français et anglais de 1902 . .	$0^s{\cdot}005$

L'extrême petitesse de cet écart est due, à n'en pas douter à une part d'heureuse chance. Mais l'erreur probable du résultat moyen par transport en avion fait bien ressortir l'ordre de précision que permet d'atteindre l'association des chronomètres actuels et des moyens de transport les plus rapides. Il est toutefois évident que les procédés basés sur la radio-télégraphie resteront dans l'avenir la meilleure méthode pour la mesure rigoureuse des différences de longitudes.

Différence de Longitudes Greenwich-Paris par Transport de Chronomètres (deck watches).

Chronomètre No.	Classement de Teddington. Ecart moyen diurne.	Ecart.	Temp.	Position.	Total Marks.	1920 Mai 18. Greenwich–Paris.	1920 Mai 20–21. Paris–Greenwich.	1920 Mai 21. Greenwich–Paris.	1920 Mai 26. Paris–Greenwich.	1920 Mai 27. Greenwich–Paris.	Moyenne par Chronomètre.
	s					m s	m s	m s	m s	m s	m s
36114	0·15	37·0	38·7	19·6	95·3	9 21·38	9 20·74	9 20·73	9 20·72	9 20·64	9 20·842
36135	0·11	37·9	38·2	19·2	95·3	9 20·93	9 20·72	9 20·96	9 20·85	9 20·82	9 20·856
36165	0·11	37·9	38·7	19·2	95·8	9 21·35	9 20·07	9 20·76	9 20·53	9 20·82	9 20·706
45065	0·11	37·9	39·6	19·4	96·9	9 20·97	9 20·61	9 20·71	9 20·82	9 20·97	9 20·816
46870	0·10	36·9	39·1	19·4	95·4	9 21·08	9 21·14	9 20·87	9 20·93	9 21·05	9 21·014
46872	0·14	37·2	36·8	18·6	92·6	9 21·21	9 20·35	9 20·90	9 20·62	9 21·01	9 20·818
46877	0·19	36·3	38·8	19·7	94·8	9 20·94	9 21·33	9 21·13	9 21·12	9 21·11	9 21·126
46878	0·11	37·9	35·2	18·2	91·3	9 21·06	9 20·56	9 20·90	9 20·72	9 20·99	9 20·846
46886	0·09	38·2	38·6	18·7	95·5	9 21·20	9 20·42	9 20·78	9 20·68	9 20·93	9 20·802
46889	0·10	38·0	38·3	18·8	95·1	9 21·01	9 21·31	9 20·93	9 20·94	9 21·12	9 21·062
46890	0·10	37·6	38·6	19·4	95·6	9 21·13	9 22·36	9 21·29	9 21·21	9 21·37	9 21·472
64974	0·10	38·0	37·9	18·9	94·8	9 21·03	9 20·97	9 20·89	9 20·95	9 21·01	9 20·970
44811						9 21·12					
Moyenne Générale	0·117	37·6	38·2	19·1	94·9	9 21·108	9 20·882	9 20·904	9 20·841	9 20·987	

Conclusions.

Moyenne générale des 61 déterminations par avion.

$9^m\ 20^s{\cdot}947$

$\pm 0^s{\cdot}027$

Différence du résultat avec la moyenne des résultats français et anglais de 1902:

$0^s{\cdot}005$

Erreur moyenne d'une détermination:

$0^s{\cdot}317$

Erreur probable d'une détermination:

$0^s{\cdot}214$

Erreur probable de la moyenne des déterminations:

$0^s{\cdot}027$

Mission de 1902.

Messrs. Dyson et Hollis:

$9^m\ 20^s{\cdot}932$

MM. Bigourdan et Lancelin:

$9^m\ 20^s{\cdot}972$

Moyenne des résultats Français et Anglais:

$9^m\ 20^s{\cdot}952$

La Chaux de Fonds.
1920 *juillet* 27.

Errata in Dr. J. Lunt's paper, pages 520 *to* 538.

p. 520. 6th line from bottom, *for* H^3 *read* H_δ^3.
p. 521. Table II., 5th line, *for* H^1 *read* H_γ^1.
p. 521. Table II., 4th line from bottom, *for* +·2 *read* +·02.
p. 522. Zero set, *for* 3998·61 *read* 3968·61.
p. 523. 1st line after 1st table, *for* width *read* widths.
p. 523. Table III., end of N_2 band, *for* 4887·6 *read* 4986·7.
p. 525. Table V., July 5, *for* 32·4 *read* 35·4.
p. 528. Line 2, *for* $-51 \pm 1{\cdot}8$ *read* $-51{\cdot}6 \pm 2{\cdot}0$.
p. 528. Table IX., end of N_2, *under* Displacement $+\lambda 0{\cdot}6$ *delete* 0·9.
p. 530. 4th line above Table XI., *for* 4910 *read* 4610.
p. 533. 2nd line, *for* 4666, 88 *read* 4711, 133.
p. 533. Footnote, *for* band *read* Bond.
p. 537. Line 28, *for* normal *read* abnormal.
p. 537. Line 37, *for* $5282\frac{3}{4}$ *read* 5282.
p. 537. Line 3 from bottom, *for* mean displacement *read* measured displacements.
p. 538. Last footnote to table, *for* H *read* H_β.

Erratum in Prof. Barnard's paper, p. 722.

5th line from bottom, *for* p. 718 *read* p. 711.

LIST OF ADDITIONS

TO THE LIBRARY

OF THE SOCIETY

JUNE 1919 TO JUNE 1920.

Serial Publications received in the regular course will, in future, not be included in this Annual List.

An asterisk (*) indicates that the work is an excerpt.

Adolpho Stahl Lectures on Astronomy:
Delivered in San Francisco, California, in 1916–1917 and 1917–1918, under the auspices of the Astronomical Society of the Pacific.
(*Purchased.*) 8vo. San Francisco, 1919

Ann Arbor, Detroit Observatory of the University of Michigan:
Publications of the Astronomical Observatory, Vol. 2.
(*Observatory.*) 4to. Ann Arbor, 1918

Bacon (Roger):
——: Opera hactenus inedita, fasc. 1–4.
(*Purchased.*) 8vo. Oxoni, 1911–1913
Fasc. 1. Metaphysica Fratris Rogeri . . . Fragmenta quæ supersunt nunc primum edidit Robert Steele.
Fasc. 2, 3. Liber primus, Communium Naturalium . . . edidit Robert Steele.
Fasc. 4. Liber secundus, Communium Naturalium . . . de Celestibus, partes quinque, edidit Robert Steele.

——: Essays contributed by various writers on the occasion of the commemoration of the seventh centenary of his birth. Collected and edited by A. G. Little.
(*Purchased.*) 8vo. Oxford, 1914

Bailey (Solon I.):
*Oliver Clinton Wendell (1845–1912).
(*Author.*) 8vo. Boston, 1919

Bakhuyzen (E. F. van de S.) and C. de Jong:
*On the influence exercised by the systematic connection between the parallax of the stars and their apparent distance from the galactic plane upon the determination of the precessional constant and of the systematic proper motions of the stars.
(*Leiden Observatory.*) 8vo. Amsterdam, 1915

Ball (John) and Harold Knox-Shaw:
A handbook of the Prismatic Astrolabe.
(*Egyptian Government.*) 8vo. Cairo, 1919

Banachiewicz (Thadeé):
——: Tables fondamentales pour la résolution de l'Equation de Gauss. 4to. Paris, 1916
——: Tables auxiliaires pour la résolution de l'Equation de Gauss.
(*Cracow Observatory.*) 8vo. Paris, 1916

Barcelona, Observatorio Fabra:
Boletín, Sección Astronómica, No. 1.
(*Observatory.*) 4to. Barcelona, 1919

Barnard (E. E.):
——: *A new Variable Star with very long period.
(*Author.*) 8vo. Northfield, Minn., 1919
——: *Variable Stars in the Cluster M11 (N.G.C. 6705).
(*Author.*) 8vo. Northfield, Minn., 1919
——: *Self-luminous night haze.
(*Author.*) 8vo. Northfield, Minn., 1919
——: *On the varnishing of astronomical negatives.
(*Author.*) 8vo. Northfield, Minn., 1919

Baxendell (Joseph):
*Observations of Variable Stars. Edited by H. H. Turner [and Mary A. Blagg]. Collection of reprints of papers in the *Monthly Notices.*
(*Prof. Turner.*) 8vo. London, 1906–1918

Benares, Mathematical Society:
Proceedings, Vol. 1, 1919.
(*Society.*) 8vo. Benares, 1920

Black (F. A.):
——: Correlation of the planets in mass and mean distance.
(*Author.*) 8vo. Inverness, 1920
——: Planetary rotation periods and group ratios: two Essays on the relations between the planets in diurnal rotation and in mass.
(*Messrs Gall & Inglis.*) 8vo. Edinburgh and London [1919]

Boccardi (Giovanni):
—— : Una pseudo theoria del pianeta Marte.
(*Author.*) 8vo. Torino, 1919
—— : Cenno riassuntivo sulla rotazione della Terra.
(*Author.*) 8vo. Torino, 1919

Boll (Franz):
Studien über Claudius Ptolemäus: ein Beitrag zur Geschichte der griechischen Philosophie und Astrologie.
(*Purchased.*) 8vo. Leipzig, 1894

Brahe (Tycho):
Tychonis Brahe Dani opera omnia, edidit J. L. E. Dreyer. Tomus 3; Tomus 4, fasciculus prior; Tomus 6.
(*Editor.*) 4to. Hauniæ, 1916–1919

Breslau, Sternwarte:
Veröffentlichungen: herausgegeben vom Direktor A. Wilkens. No. 1.
(*Observatory.*) 4to. Kiel, 1920

Breson (Elias):
Communications about observations on the RSH and STO and other "spots" on the planet Jupiter during opposition 1915–1916.
(*Author.*) Helsingfors, 1919

Brown (E. W.) and H. B. Hedrick:
Tables of the Motion of the Moon.
(*Yale University and Author.*)
3 vols. 4to. New Haven, 1919

Brussels, Académie Royale des Sciences:
—— : Tables des notices biographiques publiées dans l'Annuaire (1835–1919).
(*Academy.*) 12mo. Bruxelles, 1919
—— : Rapport succinct sur l'état du Palais des Académies après le départ des Allemands.
(*Academy.*) 8vo. Bruxelles, 1919

Buisson (H.), C. Fabry et H. Bourget:
*Application des interférences à l'étude de la nebuleuse d'Orion.
(*Authors.*) 8vo. Tours, 1914

Bulletin Astronomique. Revue générale des travaux astronomiques. 2me Série, 2me partie. Tome 1, Jan.–Juillet, 1919.
(*Editors.*) 8vo. Paris, 1919

Cambridge Observatory:
Catalogue of the Zone +25° to 30° Declination. Enlarged edition.
(*Observatory.*) 4to. Cambridge, 1919

Canada, Geodetic Survey:
Publication No. 3. Determination of the lengths of Invar Base line tapes from standard nickel bar No. 10239.
(*The Superintendent.*) 8vo. Ottawa, 1919

Cape of Good Hope, Royal Observatory:
——: Cape Astrographic Zones, Vol. 3. Catalogue of rectangular co-ordinates and diameters of star images, Zone −43°.
(*Observatory*). 4to. London, 1915
——: Fundamental Catalogue of 1293 stars from observations made during the years 1905–1911, under the direction of Sir David Gill and S. S. Hough.
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